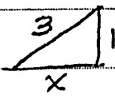


- ① shifted right $\pi/2$ & reflected over y-axis
 $\sin(\frac{\pi}{2} - \theta) = \sin(-\theta + \frac{\pi}{2}) = \sin(-(\theta - \pi/2))$

② $\sin \theta = \frac{1}{3}$; $0 < \theta < \frac{\pi}{2}$



$$\begin{aligned} 9 &= 1 + x^2 \\ \theta &= x^2 \\ x &= 2\sqrt{2} \end{aligned}$$

$$\cos \theta = \frac{2\sqrt{2}}{3}$$

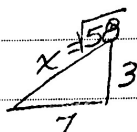
$$\tan \theta = \frac{1}{2\sqrt{2}} = \frac{1}{2\sqrt{2}} \cdot \frac{2\sqrt{2}}{2\sqrt{2}} = \frac{2\sqrt{2}}{4 \cdot 2} = \frac{\sqrt{2}}{4}$$

$$\csc \theta = 3$$

$$\sec \theta = \frac{3\sqrt{2}}{4}$$

$$\cot \theta = 2\sqrt{2}$$

③ $\tan \theta = \frac{3}{7}$; $0 < \theta < \frac{\pi}{2}$



$$x^2 = 9 + 49 = 58$$

$$\sin \theta = \frac{3}{\sqrt{58}} = \frac{3\sqrt{58}}{58}$$

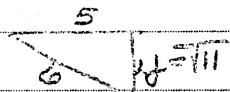
$$\cos \theta = \frac{7\sqrt{58}}{58}$$

$$\cot \theta = 7/3$$

$$\csc \theta = \sqrt{58}/3$$

$$\sec \theta = \sqrt{58}/7$$

④ $\cos \theta = \frac{5}{6}$; $\frac{3\pi}{2} < \theta < 2\pi$



$$\begin{aligned} 36 &= y^2 + 25 \\ 11 &= y^2 \end{aligned}$$

$$\sin \theta = -\frac{\sqrt{11}}{6}$$

$$\tan \theta = -\frac{\sqrt{11}}{5}$$

$$\sec \theta = 6/5$$

$$\csc \theta = -\frac{6\sqrt{11}}{11}$$

$$\cot \theta = -\frac{5\sqrt{11}}{11}$$

$$\begin{aligned} \frac{c^2 + s^2}{s^2} &= \frac{1}{s^2} \\ \frac{c^2 + s^2}{c^2} &= \frac{1}{c^2} \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} 1 + \tan^2 \theta = \sec^2 \theta$$

⑤ $\sin x \cot x = \sin x \cdot \frac{\cos x}{\sin x} = \cos x$

⑥ $\frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin \theta}{+\cos \theta} = -\tan \theta$

⑦ $\csc \theta \sin \theta + \cot^2 \theta = \frac{1}{\sin \theta} \cdot \sin \theta + \cot^2 \theta = 1 + \cot^2 \theta = \csc^2 \theta$

⑧ $\cos \theta (1 + \tan^2 \theta) = \cos \theta \cdot (\sec^2 \theta) = \cos \theta \left(\frac{1}{\cos^2 \theta} \right) = \frac{1}{\cos \theta} = \sec \theta$

$$(9) \frac{1 + \tan^2(\frac{\pi}{2} - x)}{1 + \cot^2 x}$$

$$\frac{1 + \cot^2 x}{1 + \cot^2 x}$$

$$\{ \csc^2 x \}$$

$$(10) \frac{\cos(\frac{\pi}{2} - x)}{\csc x}$$

$$\sin x \cdot \sin x$$

$$\{ \sin^2 x \}$$

$$(11) \frac{\cos(\frac{\pi}{2} - \theta)}{\csc \theta} + \cos^2 \theta$$

$$\sin \theta \cdot \sin \theta + \cos^2 \theta$$

$$\sin^2 \theta + \cos^2 \theta$$

$$\{ 1 \}$$

$$(12) \sin(\frac{\pi}{2} - \theta) \sec \theta$$

$$\cos \theta \cdot \frac{1}{\cos \theta}$$

$$\{ 1 \}$$

$$(13) \frac{\cos^2 x}{\cot^2 x}$$

$$\cos^2 x \cdot \tan^2 x$$

$$\cos^2 x \cdot \frac{\sin^2 x}{\cos^2 x}$$

$$\{ \sin^2 x \}$$

$$(14) \frac{\sec x \cdot \sin x + \cos(\frac{\pi}{2} - x)}{1 + \sec x}$$

$$\frac{\sec x \cdot \sin x + \sin x}{1 + \sec x}$$

$$\frac{\sin x (\sec x + 1)}{1 + \sec x}$$

$$\{ \sin x \}$$

$$(15) \frac{\csc^2 x - \cot^2 x}{\sin(-x) \cot x}$$

$$\frac{1 + \cot^2 x - \cot^2 x}{- \sin x \cdot \cos x / \sin x}$$

$$\frac{1}{- \cos x}$$

$$- \cos x$$

$$\{ - \sec x \}$$

$$(16) \frac{\cos^2 x \cdot \tan^2(-x) - 1}{\cos^2 x} = \cos^2 x \cdot \tan^2(-x) - \frac{1}{\cos^2 x}$$

$$\tan^2(-x) - \sec^2 x$$

$$\tan^2(-x) - (1 + \tan^2 x)$$

$$(-\tan x)(-\tan x) - 1 - \tan^2 x$$

$$\tan^2 x - 1 - \tan^2 x$$

$$\{ -1 \}$$

$$(17) \sin x \cdot \csc x = 1$$

$$\sin x \cdot \frac{1}{\sin x} = 1$$

$$1 = 1 \quad \text{✓}$$

$$(18) \tan \theta \csc \theta \cos \theta = 1$$

$$\tan \theta \left(\frac{1}{\sin \theta} \right) (\cos \theta) = 1$$

$$\frac{\sin \theta}{\cos \theta} \left(\frac{1}{\sin \theta} \right) (\cos \theta) = 1$$

$$1 = 1 \quad \text{✓}$$

$$(19) \frac{\cos(\frac{\pi}{2} - \theta) + 1}{1 - \sin(-\theta)} = 1$$

$$\frac{\sin \theta + 1}{1 - (-\sin \theta)} = 1$$

$$1 - (-\sin \theta)$$

$$\frac{\sin \theta + 1}{\sin \theta + 1} = 1$$

$$\sin \theta + 1$$

$$1 = 1 \quad \text{✓}$$

$$(20) \sin(\frac{\pi}{2} - x) \tan x = \sin x$$

$$\cos x \cdot \frac{\sin x}{\cos x} = \sin x$$

$$\sin x = \sin x \quad \text{✓}$$

(cont'd)

$$(21) \frac{\csc^2 \theta - \cot^2 \theta}{1 - \sin^2 \theta} = \sec^2 \theta$$

$$\frac{1 + \cot^2 \theta - \cot^2 \theta}{\sin^2 \theta + \cos^2 \theta - \sin^2 \theta} = \sec^2 \theta$$

$$\frac{1}{\cos^2 \theta} = \sec^2 \theta$$

$$\sec^2 \theta = \sec^2 \theta \quad \checkmark$$

$$(22) 2 - \cos^2 \theta = 1 + \sin^2 \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$2 - (1 - \sin^2 \theta) = 1 + \sin^2 \theta$$

$$2 - 1 + \sin^2 \theta = 1 + \sin^2 \theta$$

$$1 + \sin^2 \theta = 1 + \sin^2 \theta \quad \checkmark$$

$$(23) \sin x + \cos x \cot x = \csc x$$

$$\sin x + \cos x \left(\frac{\cos x}{\sin x} \right) =$$

$$\frac{\sin x}{\sin x} \cdot \sin x + \frac{\cos^2 x}{\sin x} = \csc x$$

$$\frac{\sin^2 x + \cos^2 x}{\sin x} =$$

$$\frac{1}{\sin x} = \csc x$$

$$\csc x = \csc x \quad \checkmark$$

$$(24) \frac{\sin^2(-x)}{\tan^2 x} = \cos^2 x$$

$$\frac{(-\sin x)(-\sin x)}{\tan^2 x} = \cos^2 x$$

$$\frac{\sin^2 x}{\sin^2 x / \cos^2 x} =$$

$$\cos^2 x = \cos^2 x \quad \checkmark$$

$$(25) \frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 + \cos x} = 2 \csc x$$

$$\frac{1 + \cos x}{1 + \cos x} \left(\frac{\sin x}{\sin x} \right) + \frac{\sin x}{\sin x} \left(\frac{1 + \cos x}{1 + \cos x} \right) = 2 \csc x$$

$$\frac{1 + 2 \cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)(\sin x)} = 2 \csc x$$

$$\frac{1 + 2 \cos x + 1}{(1 + \cos x)(\sin x)} =$$

$$\frac{2 + 2 \cos x}{(1 + \cos x)(\sin x)} =$$

$$\frac{2(1 + \cos x)}{(1 + \cos x)\sin x} =$$

$$2 \csc x = 2 \csc x \quad \checkmark$$

$$(26) \frac{\sin x}{1 - \cos(-x)} = \csc x + \cot x$$

$$= \frac{1}{\sin x} + \frac{\cos x}{\sin x}$$

$$= \frac{(1 + \cos x) \cdot \sin x}{\sin x \cdot \sin x}$$

$$= \frac{(1 + \cos x) \sin x}{\sin^2 x}$$

$$= \frac{(1 + \cos x)(\sin x)}{1 - \cos^2 x}$$

$$= \frac{(1 + \cos x)(\sin x)}{(1 + \cos x)(1 - \cos x)}$$

$$= \frac{\sin x}{1 - \cos x}$$

$$\frac{\sin x}{1 - \cos(-x)} = \frac{\sin x}{1 - \cos(-x)} \quad \checkmark$$