Vector Addition and Scalar Multiplication

EQ: How can I operate with a quantity that has both magnitude and direction?

MCC9-12.N.VM.4 Add and subtract vectors.

MCC9-12.N.VM.4a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

MCC9-12.N.VM.4b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

MCC9-12.N.VM.4c Understand vector subtraction v - w as v + (-w), where (-w) is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

MCC9-12.N.VM.5 Multiply a vector by a scalar.

MCC9-12.N.VM.5a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

MCC9-12.N.VM.5b Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v = 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).

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Resultant Vector - the vector that is the result of operations with vectors.

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Vector Addition and Scalar Multiplication

Let $\mathbf{u} = \langle u_1, u_2 \rangle$, and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors and let k be a scalar (a real number).

$$\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle$$

$$k\mathbf{u} = k\langle u_1, u_2 \rangle = \langle ku_1, ku_2 \rangle$$

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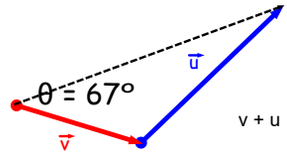
In order to add vectors, they must be head to tail, so translate one of the vectors.

u + v



v + u

Is adding vectors commutative?

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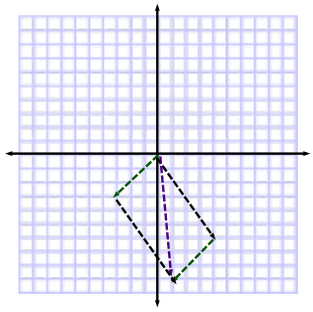

$$\langle 4, -6 \rangle + \langle -2, -3 \rangle$$

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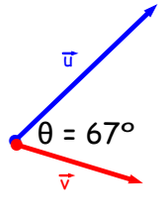
In order to subtract vectors, add the inverse of the 2nd vector head to tail.

u - v



v - u

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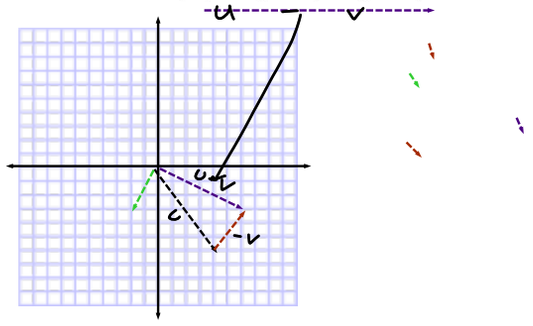
$\langle 4,-6\rangle - \langle -2,-3\rangle$



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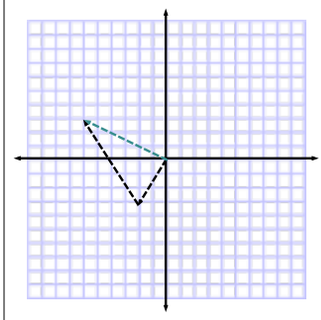
$\langle -2,-3\rangle - 1\langle 4,-6\rangle$



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$4\langle 4,-6\rangle$

$4\langle -2,-3\rangle$

$4\langle 4,-6\rangle + 4\langle -2,-3\rangle$

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Standard unit vectors

$\mathbf{i} = \langle 1,0\rangle \qquad \mathbf{j} = \langle 0,1\rangle$

$\mathbf{v} = \langle v_1, v_2\rangle = v_1\langle 1,0\rangle + v_2\langle 0,1\rangle = v_1\mathbf{i} + v_2\mathbf{j}$

v_1 and v_2 are the horizontal and vertical components respectively.

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3i - 2j + 5i + 3j

8i + j

-2i + 7j + -4i - 6j

-6i + j

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bearing = clockwise from North

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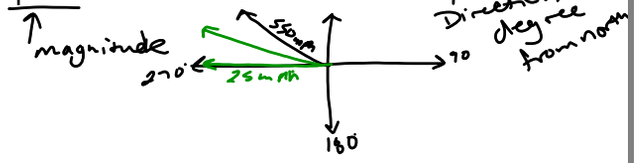
A plane is flying across the U.S. at 550 mph with a bearing of 290°. If there is a tailwind blowing due west at 25 mph, what is the bearing and speed of the resultant vector?



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Suppose that you swim across a stream that has a 5-km/hr current.

a. Find your actual velocity vector if you swim perpendicular to the current at 3 km/h.

b. Find your speed through the water if you swim perpendicular to the current but your resultant velocity makes an angle of 34° with the direction you are heading.

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