Unit 5 - Matrices

What is the purpose of this unit?	EQ: How can we use matrices to solve real life problems?
What vocab do I need?	Vocabulary: matrix, determinant, elements, dimensions, scalar, inverse matrix, identity matrix
Standards	MCC9-12.N.VM.6 Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. MCC9-12.N.VM.12 Work with 2x2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area

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Determinants

What is the purpose of this lesson?

EQ: How do we evaluate the determinants of matrices?

Vocabulary:

Determinant - a real number associated with a square matrix. The determinant of a matrix A is denoted by *det A* or *|A|*.

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A. Determinant of a 2 x 2 matrix

How do I find the determinant of a 2x2?	The determinant of a 2 x 2 matrix is the difference of the products of the elements on the diagonals.

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

In other words:

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Evaluate the determinant of the matrix

a) $\det\begin{bmatrix} 5 & 8 \\ 9 & 4 \end{bmatrix} = 5(4) - 9(8) = -52$

b) $\det\begin{bmatrix} 3 & -4 \\ 7 & -2 \end{bmatrix} = 3(-2) - 7(-4) = 22$

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B. Determinant of a 3 x 3 matrix

How do I find the determinant of a 3x3?	Rewrite the first two columns to the right of the determinant. Add the products of the leading diagonals and subtract from this the products of the opposite diagonals.

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$$

$$= (aei + bfg + cdh) - (gec + hfa + idb)$$

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Evaluate the determinant of the matrix

c) $\det\begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -2 \\ 4 & 1 & 5 \end{bmatrix}$

$= (1\cdot3\cdot5 + 2(-2)4 + (-1\cdot0\cdot1)) -$

$(4\cdot3\cdot-1 + 1\cdot-2\cdot1 + 5\cdot0\cdot2)$

$= (15 - 16 - 0) - (-12 - 2)$

$= (-1) - (-14) = 13$

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d) $\det\begin{bmatrix} 4 & 3 & 1 \\ 5 & -7 & 2 \\ 1 & -3 & 4 \end{bmatrix}$

$= (4\cdot -7\cdot 4 + 3\cdot 2\cdot 1 + 1\cdot 5\cdot -3) - (1\cdot -7\cdot 1 + -3\cdot 2\cdot 4 + 4\cdot 5\cdot 3)$

$= (-112 + 6 - 15) - (-7 - 24 + 60)$

$(-121) - (29) = -150$

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Evaluate:

e) $\begin{vmatrix} 2 & -4 \\ 3 & x \end{vmatrix} = 26$

$2x - (-4\cdot 3) = 26$

$2x + 12 = 26$

$2x = 14$

$x = 7$

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f) $\begin{vmatrix} -1 & 3 & 5 \\ 0 & x & 3 \\ -2 & 2 & x \end{vmatrix} = x - 4$

$-1\cdot x\cdot x + 3\cdot 3\cdot -2 + 5\cdot 0\cdot 2 - (-2\cdot x\cdot 5 + 2\cdot 3\cdot -1 + x\cdot 0\cdot 3) = x - 4$

$-x^2 + (-18) + (+10x + 6) = x - 4$

$-x^2 + 10x - 12 = x - 4$

$-x + 4 \quad -x + 4$

$-x^2 + 9x - 8 = 0$

$-(x^2 - 9x + 8) = 0$

$-(x - 8)(x - 1) = 0$

$x = 8 \quad x = 1$

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Application of Determinants: Finding the area of a triangle using determinants

Given the vertices of a triangle are: (x_1, y_1) (x_2, y_2), and (x_3, y_3), then its area is found by:

$$A = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

where the $\pm$ indicates the appropriate sign to yield a positive value

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1. The vertices of triangle DOG are D(- 4, - 1), O(3, 2) and G(4, 6).

Find the area of ΔDOG.

$A = \pm\frac{1}{2}\begin{vmatrix} -4 & -1 & 1 \\ 3 & 2 & 1 \\ 4 & 6 & 1 \end{vmatrix}$

$= \frac{1}{2}\left[(-4\cdot 2\cdot 1 + (-1)\cdot 1\cdot 4 + 1\cdot 3\cdot 6) - (4\cdot 2\cdot 1 + 6\cdot 1\cdot -4 + 1\cdot 3\cdot -1)\right]$

$= \pm\frac{1}{2}\left[-8 - 4 + 18 - (8 - 24 - 3)\right]$

$= \pm\frac{1}{2}(6 - (-19))$

$= \pm\frac{1}{2}(25)$

$= \boxed{\frac{25}{2}}$

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2. The vertices of triangle CAT are C(- 8, 10), A(6, 17) and T(2, - 4).

Find the area of ΔCAT.

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