Introducing Parametric Equations through Explorations   Sheet 1

Suppose two bugs are crawling along linear paths. Bug 1 begins a trek toward a point 70 inches from where he begins, traveling at a speed of 12 inches per hour. Bug 2 travels at a speed of 18 inches per hour but leaves 1 hour after the other bug from a similar starting position on a parallel path. Note: distance = rate • time (D = R • T).

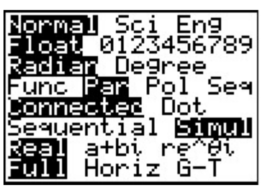
1. Given that *T* represents Bug 1's travel time, what formulas represent the distance each bug travels over time?

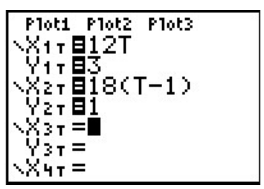
   $\text{Distance}_{\text{Bug 1}}$ = ________ • __________
   Rate   Time

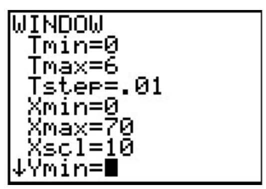
   $\text{Distance}_{\text{Bug 2}}$ = ________ • __________
   Rate   Time

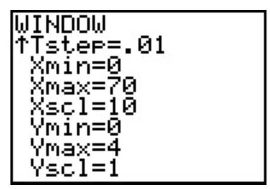
2. Which bug do you think will win the race? Why?

   Let's watch the race!









3. The Y equations are set to constants to show the bugs crawling across the screen. Why is there no *T* variable in the Y equation for each bug?

4. Graph the paths of the bugs in motion. Which bug wins the race?

5. At what time are the bugs the same distance from their starting points along their paths? In other words, when are the bugs alongside each other?

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Introducing Parametric Equations through Explorations Sheet 2

Consider a bug's nonlinear path as the bug runs around on the *xy* plane in time *t* seconds as modeled by

$$\begin{cases} x = 4\cos(t) + 5\cos(3t) \\ y = \sin(3t) + t \end{cases}$$

(Use MODE: RADIAN and PARAMETRIC)

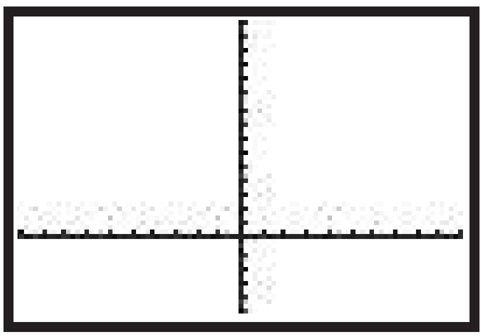
1. Where does the bug start (what are the coordinates of its initial position)?

2. Where is the bug 1 second after starting?

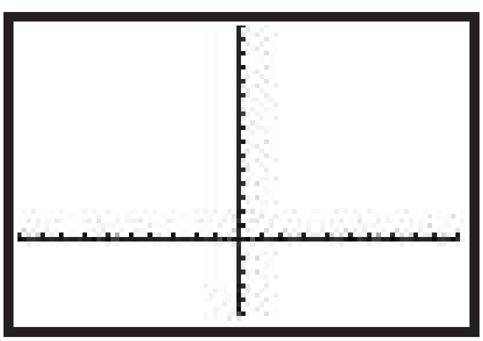
3. Build a table for the bug's location at times $t = 2, 3, 4, \ldots, 13$ seconds.

Time (sec)	Location x	Location y
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		

4. Use the table of values from question 3 to graph the bug's path from point to point, starting at $t = 0$ and ending at $t = 13$.



5. How should we adjust the graph settings if we want to see the bug's path in "slow motion" by looking at where it is every tenth of a second instead of every second? Show the new graph below.



6. How does the graph of the bug's path at every hundredth of a second compare to the graph of the bug's path at every tenth of a second?

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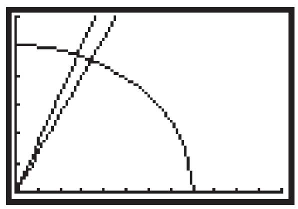
Introducing Parametric Equations through Explorations Sheet 3

Now consider *three* bugs' paths as the bugs run around on the *xy* plane in time *t* seconds as modeled by

Bug A	**Bug B**	**Bug C**
$\begin{cases} x = 8\cos(t) \\ y = 5\sin(t) \end{cases}$	$\begin{cases} x = 3t \\ y = 5t \end{cases}$	$\begin{cases} x = 3t \\ y = 4t \end{cases}$

1. Which path on the graph is Bug A's path? Which is Bug B's path? Bug C's path? Label each path in the picture with an A, B, or C, as appropriate.



If all three bugs start moving from their initial positions at the same time:

2. Do Bug A and Bug B collide? Justify your answer.

3. Do Bug B and Bug C collide? Justify your answer.

4. Do Bug A and Bug C collide? Justify your answer.