Name: ________________________

Vectors in the City Learning Task

Amy is spending some time in a city that is laid out in square blocks. The blocks make it very easy to get around so most directions are given in terms of the number of blocks you need to walk and the direction you need to go. Amy had a series of places she wanted to go to when she left the hotel on Monday morning. The concierge had helped her map out the directions. The directions looked like this:

Stop 1	3 East	5 North
Stop 2	5 East	2 North
Stop 3	2 East	1 North

1. Use a piece of graph paper and record Amy's trip. Place her hotel at the origin.

2. How many total blocks East did Amy walk?

3. How many total blocks North did Amy walk?

4. Add directions to the chart for Amy to walk so she can get back to the hotel in only one turn.

5. A very tired Amy got back to the hotel and wished she could have walked directly back to the hotel from her last stop. How much shorter would it have been if Amy could have walked directly back?

6. The next morning she was ready to go again. She had even more stops this time. On Tuesday her directions looked like this.

Stop 1	2 West	0 North
Stop 2	0 West	4 South
Stop 3	3 West	1 South
Stop 4	5 West	1 South
Stop 5	0 West	2 South

7. Amy walked back to the hotel only making one turn. She was surprised that it took her the same amount of time to walk back as it had on Monday. Add her last set of directions to return to hotel to the chart above.

Is there a reason that it took her the same amount of time to walk back?

8. Was her shortest distance back to the hotel the same on Monday and Tuesday?

9. On Wednesday her path was going to be taking her all over the place. Map out her directions on a grid.

Stop 1	2 East	3 North
Stop 2	4 West	2 North
Stop 3	3 East	1 South
Stop 4	5 SOUTHWEST* (4 West and 3 South)	
Stop 5	3 East	1 South

*go further west than south

10. Amy was amazed to find herself back at the hotel when she finished on Wednesday. Did you end up back at the hotel on your grid? If not, go back and check your SouthWest direction. Think back to the Pythagorean Theorem to determine where she ends up in 5 blocks.

11. Look back at the directions for Monday and Tuesday. What do you notice about the sums of the blocks as she made her stops and the directions you added to get her back to the hotel in just one turn?

12. What happens if you add the blocks she walked on Wednesday? Do you get zero? What if you replace the 5 blocks southwest with 3 blocks south and 4 blocks west? What do you need to do about the signs of the numbers to get it to sum to zero?

13. How are the directions North, South, East and West related to the coordinate plane?

14. Where is NE? SE? NW? SW? Draw a coordinate plane and determine the <u>equation of a line</u> traveling in each of those directions.

15. What would NNE mean? What would it look like on your coordinate plane? Can you identify at least two coordinates that lie on a line pointed in the NNE direction?

NNE would bisect the angle between N and NE.

If Amy were not walking around a city where buildings did not block her path, it would be a shorter walk for her if she could walk directly to the position indicated by the two directions given to her at the hotel.

16. Determine a single set of directions that could get Amy to a point that is 3 blocks East and 4 blocks north of her hotel. Note that her direction is not exactly NE. Determine another way to define her direction from the hotel.

17. Did you determine a distance and a direction for Amy to walk? How did you describe her direction?

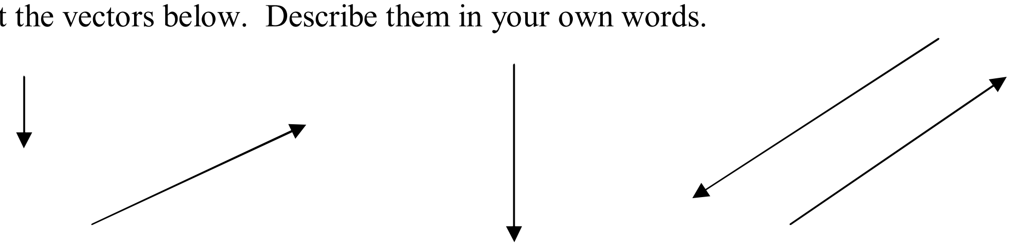
In mathematics we use directed line segments, or **vectors**, to indicate a magnitude (length or distance) and a direction.

In Amy's situation, using the Pythagorean Theorem helps us find the magnitude of 5. To describe the direction, we use the angle the vector makes with the *x*-axis. The angles are measured in the same way angles are measured on the unit circle: counter-clockwise is positive, clockwise is negative.

18. The specific direction Amy traveled can be found using $\theta = \tan^{-1} \frac{4}{3}$. Is that what you did? If not, do it now and determine the direction she was traveling.

The length of the directed line segment is used to represent the magnitude of a vector. The direction the segment points in is indicative of the direction of the vector.

19. Look at the vectors below. Describe them in your own words.

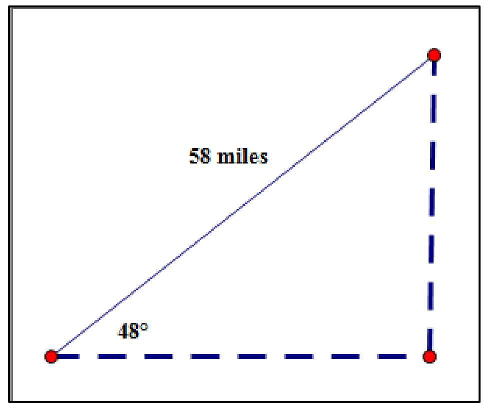


The directions given to Amy were the **horizontal and vertical components** of a vector. Is it possible to determine the magnitude and direction of the vector using the components of the vector?

20. A pilot traveled due East for 3 miles and then turned due North for 8 miles. Write a single vector to describe his distance and direction from his base.

At other times we want to take a single vector apart and look at its components. This is possible using geometry and trigonometry.

21. A ship leaves port and sails 58 miles in a direction 48° North of due East. Find the magnitude of the vertical and horizontal components. (The drawing should help.)



22. A plane flies 150 miles 38 ° south of due West. Draw a diagram of the flight and determine the horizontal and vertical components of the flight.

Often, there are two vectors that combine to describe a third vector. The sum of two or more vectors is called the resultant vector. There are several ways to find the resultant vector.

23. Draw a diagram to represent the following problem. A ship leaves port and travels 49 miles at a direction of 30° . The ship then turns an additional 40° north of due east and travels for 89 miles. At that point the ship drops anchor. A helicopter needs to join the ship. If the helicopter is leaving the same port, what vector should be reported to the pilot that describes a direct path to the plane? (Hint: Draw a diagram. Create two right triangles to find the total horizontal and vertical distances.)

24. Vectors are also used to represent forces in physics. Two forces can act together or against each other. Either way, one force will have a definite effect when combined with another force.

- **a.** Imagine two people pulling on your arms with the same amount of force, both in a northerly direction (picture yourself facing north). What effect would they have?
- **b.** What if one person pulls towards the north while the other pulls towards the south?
- **c.** What if the person pulling towards the north is stronger than the one pulling south? How would that effect you?

When two or more vectors are added together, the resultant vector can be found by summing the horizontal and vertical components of each vector.

25. You jump into a river intending to swim straight across to the other side. When you started swimming you realized the current was stronger than you expected. In fact, the current was pushing you directly south at 4 miles/hour. You were swimming directly East at 1 mile/hour. If you do not change your direction, where will you land when you touch the other side 15 minutes later?

26. A plane traveling at 400 mph is flying with a bearing (measured from due North) of 40°. There is a wind of 50 mph from the South. If no correction is made for the wind, what are the final bearing and ground speed of the plane?

27. Vectors can also be represented algebraically using ordered pairs. If vector **w** starts at the origin and ends at the point (-4, 6), determine the magnitude and direction of the vector. Vector **w** is written as $\vec{w} = \langle -4, 6 \rangle$.

28. If adding vectors written algebraically, add each of the corresponding terms to get the resultant vector. Similarly, if you are to subtract the vector, subtract the corresponding terms. Multiplying a vector is called scalar multiplication. Scalar multiplication multiplies each part of the vector.

Let $\vec{w} = \langle -2, 3 \rangle$, $\vec{a} = \langle 4, 10 \rangle$, $\vec{m} = \langle -4.2, -8 \rangle$, $\vec{k} = \langle 0.5, 22 \rangle$

$\vec{w} + \vec{k} = \langle -2 + 0.5, 3 + 22 \rangle = \langle -1.5, 25 \rangle$ $\quad$ $\vec{a} - \vec{m} = \langle 4 - (-4.2), 10 - (-8\_ \rangle = \langle 0.2, 18 \rangle$

$3\vec{w} + 2\vec{a} = 3\langle -2, 3 \rangle + 2\langle 4, 10 \rangle = \langle -6, 9 \rangle + \langle 8, 20 \rangle = \langle 2, 29 \rangle$

Complete the following:

$4\vec{a} - 5\vec{k} + \vec{k} =$ ____

Notes on Vectors in the City Learning Task

Supplies:

- ***Graphing calculator***
- ***Graph paper***

This task is designed to introduce students to vectors through a simple problem situation involving walking around a city that is laid out in blocks. The idea is to ground them in the understanding that a vector can always be broken into horizontal and vertical components. Since that is what they are most familiar with we start with those components and move into the vector components of magnitude and direction.

Vectors in the City Learning Task

Amy is spending some time in a city that is laid out in square blocks. The blocks make it very easy to get around so most directions are given in terms of the number of blocks you need to walk and the direction you need to go. Amy had a series of places she wanted to go to when she left the hotel on Monday morning. The concierge had helped her map out the directions. The directions looked like this:

Stop 1	3 East	5 North
Stop 2	5 East	2 North
Stop 3	2 East	1 North
Return	***10 West***	***8 South***

29. Use a piece of graph paper and record Amy's trip. Place her hotel at the origin.

30. How many total blocks East did Amy walk?

She walked 10 blocks east.

31. How many total blocks North did Amy walk?

She walked 8 blocks north.

32. Add directions to the chart for Amy to walk so she can get back to the hotel in only one turn.

See table above

33. A very tired Amy got back to the hotel and wished she could have walked directly back to the hotel from her last stop. How much shorter would it have been if Amy could have walked directly back?

She would have walked the hypotenuse of a right triangle with sides of 10 and 8. Using Pythagorean Theorem:

$$d^2 = 10^2 + 8^2$$
$$d = 12.8\ blocks$$

She walked 18 blocks so this would have saved her a distance of 5.2 blocks.

34. The next morning she was ready to go again. She had even more stops this time. On Tuesday her directions looked like this.

Stop 1	2 West	0 North
Stop 2	0 West	4 South
Stop 3	3 West	1 South
Stop 4	5 West	1 South
Stop 5	0 West	2 South
Return	***10 East***	***8 North***

35. Amy walked back to the hotel only making one turn. She was surprised that it took her the same amount of time to walk back as it had on Monday. Add her last set of directions to return to hotel to the chart above.

Is there a reason that it took her the same amount of time to walk back?

The distances add up to the same number of blocks. She just went in different directions.

36. Was her shortest distance back to the hotel the same on Monday and Tuesday?

Yes. The right triangle is the same size.

37. On Wednesday her path was going to be taking her all over the place. Map out her directions on a grid.

Stop 1	2 East	3 North
Stop 2	4 West	2 North
Stop 3	3 East	1 South
Stop 4	5 SOUTHWEST*	
Stop 5	3 East	1 South

*go further west than south

38. Amy was amazed to find herself back at the hotel when she finished on Wednesday. Did you end up back at the hotel on your grid? If not, go back and check your SouthWest direction. Think back to the Pythagorean Theorem to determine where she ends up in 5 blocks.

Answers will vary. The key is the 5 blocks SW is the same as 3 blocks south and 4 blocks west.

39. Look back at the directions for Monday and Tuesday. What do you notice about the sums of the blocks as she made her stops and the directions you added to get her back to the hotel in just one turn.

Answers will vary. Students need to realize the final directions are just sums of the earlier directions.

40. What happens if you add the blocks she walked on Wednesday? Do you get zero? What if you replace the 5 blocks southwest with 3 blocks south and 4 blocks west? What do you need to do about the signs of the numbers to get it to sum to zero?

Answers will vary. West and South should be treated as negative directions to get a sum of zero.

41. How are the directions North, South, East and West related to the coordinate plane?

North is the positive y direction.
East is the positive x direction.
South is the negative y direction.
West is the negative x direction.
Lengths are not negative. The negative implies direction.

42. Where is NE? SE? NW? SW? Draw a coordinate plane and determine the coordinates of a line traveling in each of those directions.

NE, SE, NW and SW are the angle bisectors of each quadrant.
They are 45° from the x and y axes.
They can be defined by the equations $y = x$ and $y = -x$.

43. What would NNE mean? What would it look like on your coordinate plane? Can you identify at least two coordinates that lie on a line pointed in the NNE direction?

NNE would bisect the angle between N and NE.

If Amy were not walking around a city where buildings did not block her path, it would be a shorter walk for her if she could walk directly to the position indicated by the two directions given to her at the hotel.

44. Determine a single set of directions that could get Amy to a point that is 3 blocks East and 4 blocks North of her hotel. Note that her direction is not exactly NE. Determine another way to define her direction from the hotel.

Walk 5 blocks at an angle of 53° from the x-axis.

45. Did you determine a distance and a direction for Amy to walk? How did you describe her direction?

In terms of the angle formed at the origin.

In mathematics we use directed line segments, or **vectors**, to indicate a magnitude (length or distance) and a direction.

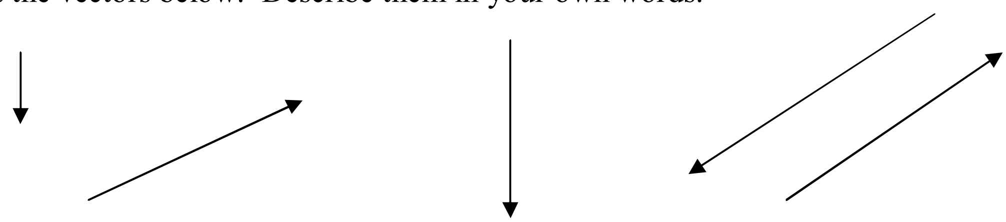
In Amy's situation, using the Pythagorean Theorem helps us find the magnitude of 5. To describe the direction, we use the angle the vector makes with the x-axis. The angles are measured in the same way angles are measured on the unit circle: counter-clockwise is positive, counter clockwise is negative.

46. The specific direction Amy traveled can be found using $\theta = \tan^{-1}\frac{4}{3}$. Is that what you did? If not, do it now and determine the direction she was traveling.

Answers will vary based on what students did.

The length of the directed line segment is used to represent the magnitude of a vector. The direction the segment points in is indicative of the direction of the vector.

47. Look at the vectors below. Describe them in your own words.

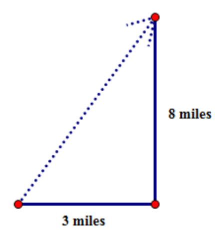


Answers will vary. Things to notice: one vector is shorter than all the other. They are pointing in different directions. Two are point straight down. Two seem to be the same length but are going in opposite directions.

The directions given to Amy were the **horizontal and vertical components** of a vector. It is possible to determine the magnitude and direction of the vector using the components of the vector.

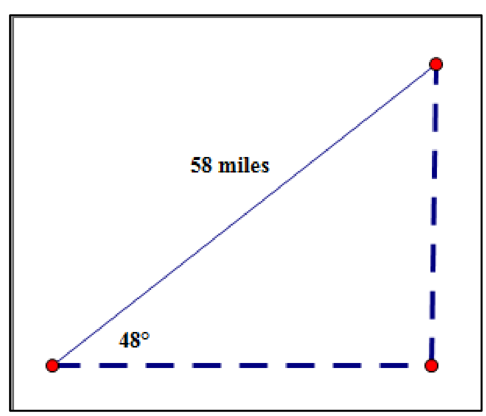
48. A pilot traveled due East for 3 miles and then turned due North for 8 miles. Write a single vector to describe his distance and direction from his base.

The distance he traveled is found using Pythagorean Theorem: $d = 8.54$ miles
The angle is found using the inverse tangent ratio: $\theta = 69.4°$
He can travel 8.54 miles at an angle of 69.4°.



At other times we want to take a single vector apart and look at its components. This is possible using geometry and trigonometry.

49. A ship leaves port and sails 58 miles in a direction 48° North of due East. Find the magnitude of the vertical and horizontal components. (The drawing should help.)

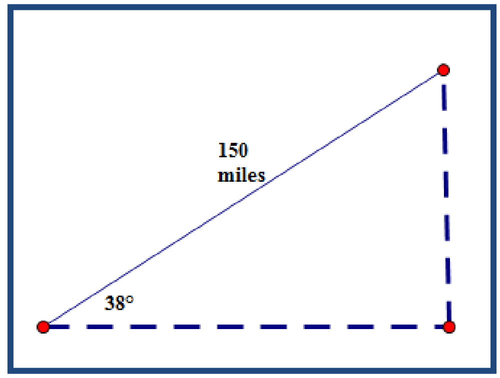


Comment:
Student can label the point representing the ship's current location as (38.8, 43.1) which was found by multiplying (58cos48, 58sin48). Make sure to discuss this with students in subsequent problems.

Solution:
Let x=horizontal magnitude and y=vertical magnitude.

$\cos 48° = x/58$	$\sin 48° = y/58$
$x = 58\cos 48$	$y = 58\sin 48$
$x = 38.8$ miles	$y = 43.1$ miles

50. A plane flies 150 miles 38 ° south of due West. Draw a diagram of the flight and determine the horizontal and vertical components of the flight.



Let x=horizontal magnitude and y=vertical magnitude.

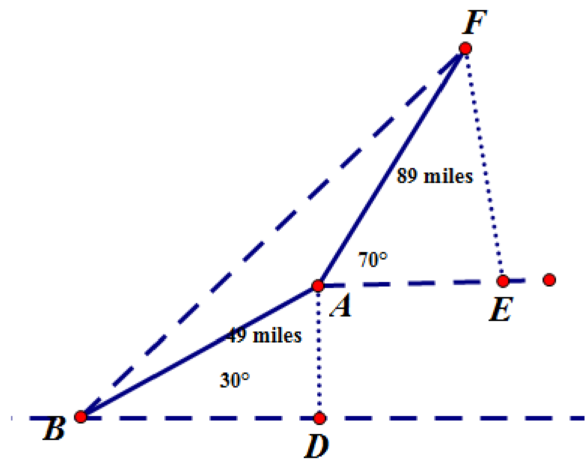
cos 38°=x/150	***sin 38° = y/150***
x = 150 cos 48	***y= 150 sin48***
x = 118.2 miles	***y = 92.3 miles***

Often, there are two vectors that combine to describe a third vector. The sum of two or more vectors is called the resultant vector. There are several ways to find the resultant vector.

51. Draw a diagram to represent the following problem. A ship leaves port and travels 49 miles at a direction of 30° . The ship then turns an additional 40° north of due east and travels for 89 miles. At that point the ship drops anchor. A helicopter needs to join the ship. If the helicopter is leaving the same port, what vector should be reported to the pilot that describes a direct path to the plane? (Hint: Draw a diagram. Create two right triangles to find the total horizontal and vertical distances.)

BD = 49 cos 30 = 42.44 ***AD = 49 sin 30= 24.5***
AE = 89 cos 70 = 30.43 ***FE = 89 sin 70 = 83.6***

Total horizontal movement = 72.78
Total vertical movement = 108.1
D = 130.32 (using Pythagorean Theorem)
Angle B = inverse tan (108.2/72.78) = 56.07°



52. Vectors are also used to represent forces in physics.
Two forces can act together or against each other. Either way, one force will have a definite effect when combined with another force.

a. Imagine two people pulling on your arms with the same amount of force, both in a northerly direction (picture yourself facing north). What effect would they have?

You will be pulled north.

b. What if one person pulls towards the north while the other pulls towards the south?

You will not be moved

c. What if the person pulling towards the north is stronger than the one pulling south? How would that effect you?

You will be moved north, but not as with as much force as only the person pulling north would do alone.

When two or more vectors are added together, the resultant vector can be found by summing the horizontal and vertical components of each vector.

53. You jump into a river intending to swim straight across to the other side. When you started swimming you realized the current was stronger than you expected. In fact, the current was pushing you directly south at 4 miles/hour. You were swimming directly East at 1 mile/hour an hour. If you do not change your direction, where will you land when you touch the other side 15 minutes later?

1 mile downstream

54. Since you do not want to land so far downriver you decide to swim at a direction of 35° north of due east. How far downstream will you now land?

.8566 miles downstream

55. A plane traveling at 400 mph is flying with a bearing of 40°. There is a wind of 50 mph from the South. If no correction is made for the wind, what are the final bearing and ground speed of the plane?

Bearing: 35.8062° Ground speed: 439.4790 mph

56. Vectors can also be represented algebraically using ordered pairs. If vector w starts at the origin and ends at the point -4, 6, determine the magnitude and direction of the vector. Vector w is written as $\vec{w} = \langle -4, 6 \rangle$.

Magnitude: 7.21 Direction: 56.3° North of due West

57. If adding vectors written algebraically, add each of the corresponding terms to get the resultant vector. Similarly, if you are to subtract the vector, subtract the corresponding terms. Multiplying a vector is called scalar multiplication. Scalar multiplication multiplies each part of the vector.

Let $\vec{w} = \langle -2, 3 \rangle$, $\vec{a}\ \langle 4, 10 \rangle =$, $\vec{m} = \langle -4.2, -8 \rangle$, $\vec{k} = \langle 0.5, 22 \rangle$

$\vec{w} + \vec{k} = \langle -2 + 0.5, 3 + 22 \rangle = \langle -1.5, 25 \rangle$ $\quad \vec{a} - \vec{m} = \langle 4 - (-4.2), 10 - (-8) \rangle = \langle 0.2, 18 \rangle$

$3\vec{w} + 2\vec{a} = 3\langle -2, 3 \rangle + 2\langle 4, 10 \rangle = \langle -6, 9 \rangle + \langle 8, 20 \rangle = \langle 2, 29 \rangle$

Complete the following:

$4\vec{a} - 5\vec{k} + \vec{k} =$ _$\langle 14, -48 \rangle$_______________________________________

An interactive website to use with vectors is:
http://www.geogebra.org/en/upload/files/english/jwelker/vector_projection.html