CCGPS Precalculus Name_________________________
Unit 5 Matrices
Operations on Matrices Learning Task Date__________________Block___

EQ: How do we perform operations on matrices?
How do we multiply matrices?
How do the properties of matrices compare with the properties of real numbers?

MCC9-12.N.VM.6 (+) Use matrices to represent and manipulate data
MCC9-12.N.VM.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
MCC9-12.N.VM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.

Introduction: Matrices allow us to perform many useful mathematical tasks which ordinarily require a large number of computations. Some types of problems which can be done efficiently with matrices include solving systems of equations, finding the area of triangles given the coordinates of the vertices, finding equations for graphs given sets of ordered pairs, and determining information contained in vertex edge graphs. In order to address these types of problems, it is necessary to understand more about matrix operations and properties; and, to use technology to perform some of the computations.

Matrix operations have many of the same properties as real numbers. There are more restrictions on matrices than on real numbers, however, because of the rules governing matrix addition, subtraction, and multiplication. Some of the real number properties which are more useful when considering matrix properties are listed below.

Let a, b, and c be real numbers	ADDITION PROPERTIES	MULTIPLICATION PROPERTIES
COMMUTATIVE	a + b = b + a	ab = ba
ASSOCIATIVE	(a + b) + c = a + (b + c)	(ab)c = a(bc)
IDENTITY	There exists a unique real number zero, 0, such that a + 0 = 0 + a = a.	There exists a unique real number one, 1, such that a * 1 = 1 * a = a
INVERSE	For each real number a, there is a unique real number - a such that a + (-a) = (-a) + a = 0	For each nonzero real number a, there is a unique real number $\frac{1}{a}$ such that $a(\frac{1}{a}) = (\frac{1}{a})a = 1$

Central High School Booster Club: In order to raise money for the school, the Central High School Booster Club offered spirit items prepared by members for sale at the school store and at games. They sold stuffed teddy bears dressed in school colors, tote bags and tee shirts with specially sewn and decorated school insignias.

The teddy bears, tote bags, and tee shirts were purchased from wholesale suppliers and decorations were cut, sewn and painted, and attached to the items by booster club parents. The wholesale cost for each teddy bear was $4.00, each tote bag was $3.50 and each tee shirt was $3.25. Materials for the decorations cost $1.25 for the bears, $0.90 for the tote bags and $1.05 for the tee shirts.

Parents estimated the time necessary to complete a bear was 15 minutes to cut out the clothes, 20 minutes to sew the outfits, and 5 minutes to dress the bears. A tote bag required 10 minutes to cut the materials, 15 minutes to sew and 10 minutes to glue the designs on the bag. Tee shirts were made using computer generated transfer designs for each sport which took 5 minutes to print out, 6 minutes to iron on the shirts, and 20 minutes to paint on extra detailing.

The booster club parents made spirit items at three different work meetings and produced 30 bears, 30 tote bags, and 45 tee shirts at the first session. 15 bears, 25 tote bags, and 30 tee shirts were made during the second meeting; and, 30 bears, 35 tote bags and 75 tee shirts were made at the third session.

They sold the bears for $12.00 each, the tote bags for $10.00 each and the tee shirts for $10.00 each. In the first month of school, 10 bears, 15 tote bags, and 50 tee shirts were sold at the bookstore. During the same time period, Booster Club members sold 50 bears, 20 tote bags, and 100 tee shirts at the games.

REFERENCES: Crisler, N., Fisher, P., & Froelich, G. (1994). *Discrete mathematics through Applications*. New York: W.H. Freeman and Company. Department of Mathematics and Computer Science; North Carolina School of Science and Mathematics. (2000). *Contemporary Precalculus through Applications, 2nd edition*. Chicago: Everyday Learning Corporation.

Part I

Vocabulary: The following is a **matrix**, a rectangular array of discrete data values, showing the wholesale cost of each item as well as the cost of decorations. "*wholesale*" and "*decorations*" are labels for the matrix **rows** and "*bears*", "*totes*", and "*shirts*" are labels for the matrix **columns**. The **dimensions of this matrix** called **A** are 2 rows and 3 columns and matrix **A** is referred to as a **[2 x 3] matrix**. Each number in the matrix is called an **entry**.

$$\textit{Cost per Item}$$

$$A = \begin{array}{r} \\ \textit{wholesale} \\ \textit{decorations} \end{array} \begin{array}{c} \begin{array}{ccc} \textit{bears} & \textit{totes} & \textit{shirts} \end{array} \\ \begin{bmatrix} 4.00 & 3.50 & 3.25 \\ 1.25 & .90 & 1.05 \end{bmatrix} \end{array}$$

It is sometimes convenient to write matrices (plural of matrix) in a simplified format without labels for the rows and columns. Matrix A can be written as an array.

$A = \begin{bmatrix} 4.00 & 3.50 & 3.25 \\ 1.25 & .90 & 1.05 \end{bmatrix}$ where the values can be identified as $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$. In this system, the entry a_{22} = .90, which is the cost of decorations for tote bags.

Q1 Write and label matrices for the information given on the Central High School Booster Club's spirit project.

a. Let matrix B show the information given on the time necessary to complete each task for each item. Labels for making the items should be *cut/print*, *sew/paint*, and *attach/dress*.

B =

b. Find matrix C to show the numbers of bears, totes, and shirts produced at each of the three meetings.

C =

c. Matrix D should contain the information on items sold at the bookstore and at the game.

D =

d. Let matrix E show the sales prices of the three items.

E =

Square matrices occur when the number of rows = the number of columns. A matrix with only one row or only one column is called a **row matrix** or a **column matrix**.

Q2 Are any of the matrices from **Q1** square matrices or row matrices or column matrices? If so, identify them.

Part II

Addition and Subtraction: Matrices can be added and subtracted. In order to **add or subtract two matrices**, the matrices must have the same dimensions. And, if the matrices have row and column labels, these labels must also match. Consider the following problem and matrices.

EX: $\begin{bmatrix} 2 & -4 & 3 \\ 6 & 1 & -5 \end{bmatrix} + \begin{bmatrix} 2 & 5 & 4 \\ -4 & -3 & 3 \end{bmatrix} =$

Q3 Several local companies wish to donate spirit items which can be sold along with the items made by the Booster Club at games help raise money for Central High School. J J's Sporting Goods store donates 100 caps and 100 pennants in September and 125 caps and 75 pennants in October. Friendly Fred's Sporting store donates 105 caps and 125 pennants in September and 110 caps and 100 pennants in October.

a. Write a matrix to describe the number of caps and pennants the JJ's Sporting Goods store donates.

JJ =

b. Write a matrix to describe the number of caps and pennants the Friendly Fred's Sporting Goods store donates.

FF =

c. Add the matrices to find out how many items are available each month from both sources.

Q4 The construction of matrix $\begin{array}{cccc} & bears & totes & shirts \\ G = \cos t & [\$5.25 & \$4.40 & \$4.30] \end{array}$ corresponds to production cost per item. Use matrix G and matrix E from #1 to find matrix P, the profit the Booster Club can expect from the sale of each bear, tote bag, and tee shirt.

Enduring Understandings

Q5 When can two matrices be added or subtracted?

Q6 How do you add or subtract two matrices?

Q7 When matrices are added or subtracted, how does the dimensions of the resulting matrix compare with the dimensions of the original matrices?

Part III

Scalar Multiplication: Matrices can be multiplied by any real number (called a scalar).

EX: $3A = 3 \bullet \begin{bmatrix} 2 & -4 & 3 \\ 6 & 1 & -5 \end{bmatrix} =$

Q8 Use scalar multiplication to change matrix B (from **Q1(a)**) from minutes required per item to hours required per item.

Matrix Multiplication:

Two matrices can be multiplied if

The resulting matrix is

Use only the rows from the first matrix.

$$\begin{bmatrix} 2 & -3 & 5 \\ -1 & -4 & 12 \end{bmatrix} \bullet \begin{bmatrix} -5 & 7 & 1 \\ 3 & 0 & 2 \\ -8 & 10 & -6 \end{bmatrix}$$

Use only the columns from the second matrix.

Multiply **by** **to get**

Then multiply **by** **to get**

Repeat this process to complete the resulting matrix.

$$\begin{bmatrix} 2 & -3 & 5 \\ -1 & -4 & 12 \end{bmatrix} \bullet \begin{bmatrix} -5 & 7 & 1 \\ 3 & 0 & 2 \\ -8 & 10 & -6 \end{bmatrix} = \begin{bmatrix} \_\_\_\_\_\_\_\_ & \_\_\_\_\_\_\_\_ & \_\_\_\_\_\_\_\_ \\ \_\_\_\_\_\_\_\_ & \_\_\_\_\_\_\_\_ & \_\_\_\_\_\_\_\_ \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 5 \\ -1 & -4 & 12 \end{bmatrix} \bullet \begin{bmatrix} -5 & 7 & 1 \\ 3 & 0 & 2 \\ -8 & 10 & -6 \end{bmatrix} = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

Q9 At the beginning of November a stomach virus hits Central High School. Students in the freshman and sophomore classes are well, a little sick, or really sick. The following tables show freshmen and sophomores according to their levels of sickness and their gender.

Student Population

Categories	Male	Female
Freshmen	250	300
Sophomores	200	275

% of Sick Students

Categories	Freshmen	Sophomores
Well	20%	25%
Little Sick	50%	40%
Really Sick	30%	35%

Suppose school personnel needed to prepare a report and include the total numbers of well and sick freshmen and sophomores in the school. Use matrix multiplication to help prepare the report.
(Hint: [level of sickness x class] * [class x gender] = [level of sickness x gender])

Q10 $S = \begin{array}{c|cc} & male & female \\ \hline well & 60\% & 70\% \\ sick & 40\% & 30\% \end{array}$ $\qquad C = \begin{array}{c|cc} & male & female \\ \hline Jr & 150 & 210 \\ Sr & 100 & 50 \end{array}$

(Hint: Sometimes it is necessary to exchange the rows and columns of a matrix to make it possible to multiply. This is called finding the **transpose** of a matrix and is most useful with labeled matrices.)

Find SC^T:

Part IV

Matrix Equations: Simplify each side if necessary so there is only one matrix on each side of the equal sign. Set corresponding elements equal and solve for the variables.

EX: $\begin{bmatrix} 4x+5 & 9 \\ 7 & -2y+3 \end{bmatrix} = \begin{bmatrix} 21 & 9 \\ 7 & y-12 \end{bmatrix}$

Enduring Understandings

Q11 When can a matrix be multiplied by a scalar?

Q12 When can two matrices be multiplied?

Q13 When matrices are multiplied, how does the dimensions of the resulting matrix compare with the dimensions of the original matrices?

Simplify. Write "undefined" for expressions that are undefined.

1) $\begin{bmatrix} -2 & 2 & 3 \\ -2 & 5 & -4 \end{bmatrix} + \begin{bmatrix} -2 & 6 & -3 \\ 2 & -1 & -5 \end{bmatrix}$

2) $\begin{bmatrix} -2 & 4 & -4 \\ 3 & 4 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 3 & 0 \\ -6 & 4 & 4 \end{bmatrix}$

3) $\begin{bmatrix} 5 \\ -1 \\ 6 \\ -6 \end{bmatrix} - \begin{bmatrix} -1 \\ -5 \\ -2 \\ -3 \end{bmatrix}$

4) $\begin{bmatrix} 2 & 4 & -1 & -2 \\ -1 & -3 & -6 & -2 \end{bmatrix} - \begin{bmatrix} 0 & 3 & -6 & 0 \\ 3 & -1 & 2 & -3 \end{bmatrix}$

5) $\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 2 & 5 \end{bmatrix}$

6) $\begin{bmatrix} 1 & -6 \\ -5 & 0 \\ -5 & 4 \end{bmatrix} - \begin{bmatrix} -3 & -3 \\ -5 & -1 \\ -3 & -6 \end{bmatrix}$

7) $\begin{bmatrix} -5 & 1 & 1 \\ 5 & 5 & -4 \end{bmatrix} - \begin{bmatrix} 5 & 5 & 2 \\ 0 & -4 & 3 \end{bmatrix}$

8) $\begin{bmatrix} -5 \\ 3 \\ 6 \end{bmatrix} - \begin{bmatrix} -1 \\ -5 \\ 6 \end{bmatrix}$

9) $\begin{bmatrix} -1 & 6 \\ 5 & 2 \\ -5 & -1 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} -3 & 5 \\ 4 & 1 \\ 1 & -1 \\ 4 & -4 \end{bmatrix}$

10) $\begin{bmatrix} 4 & -2 & 3 & 4 \end{bmatrix} + \begin{bmatrix} -6 & 2 & -3 & 4 \end{bmatrix}$

11) $\begin{bmatrix} 0 & -5 & -3 & -4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 4 & -1 \end{bmatrix}$

12) $\begin{bmatrix} 5 & 5 & -2 & 2 \\ -1 & 6 & 5 & -6 \end{bmatrix} - \begin{bmatrix} 1 & 1 & -2 & -2 \\ -1 & 3 & -2 & -6 \end{bmatrix}$

Decide if products AB and BA are possible. State the dimensions of the solution matrix of each.

1. A: 3x3, B: 3x1
2. A: 2x3, B: 2x3
3. A: 3x1, B: 1x3
4. A: 3x3, B: 1x3
5. A: 2x2, B: 2x2

Find the product. If not defined, state the reason.

6. $\begin{bmatrix} 1 & 4 \\ -2 & 7 \end{bmatrix}\begin{bmatrix} -1 & 0 & 3 \\ -2 & 4 & 1 \end{bmatrix}$

7. $\begin{bmatrix} 4 & x & -4 \end{bmatrix}\begin{bmatrix} x \\ 6 \\ 11 \end{bmatrix}$

8. $\begin{bmatrix} -1 & z \\ 9 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 & 8 \\ 7 & -3 & z \\ 4 & 1 & 0 \end{bmatrix}$

9. $\begin{bmatrix} 6 & -8 \\ 3 & 5 \\ 0 & k \end{bmatrix}\begin{bmatrix} -2 & 0 & k \\ -5 & 11 & 2 \end{bmatrix}$

10. $\begin{bmatrix} 1 \\ 4 \end{bmatrix}\begin{bmatrix} 5 & 3 & 4 \end{bmatrix}$

11. $\begin{bmatrix} 3 & 10 \\ 8 & -5 \end{bmatrix}\begin{bmatrix} -2 & 9 \\ 5 & -3 \end{bmatrix}$

12. $\begin{bmatrix} 3 & -7 & 6 \\ 11 & -4 & 0 \end{bmatrix}\begin{bmatrix} 2 & -a & 1 \\ a & -2 & -5 \end{bmatrix}$

13. $\begin{bmatrix} \frac{1}{2} & -1 \\ 2 & \frac{1}{4} \end{bmatrix}\begin{bmatrix} 0 & \frac{3}{4} \\ 3 & \frac{-1}{4} \end{bmatrix}$

14. $\begin{bmatrix} 0.2 & 1.4 \\ 0.4 & 1.5 \end{bmatrix}\begin{bmatrix} -0.3 & 2.1 \\ 0.5 & 2.2 \end{bmatrix}$

15. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \frac{1}{3} & -2 \\ 5 & \frac{2}{6} \end{bmatrix}$