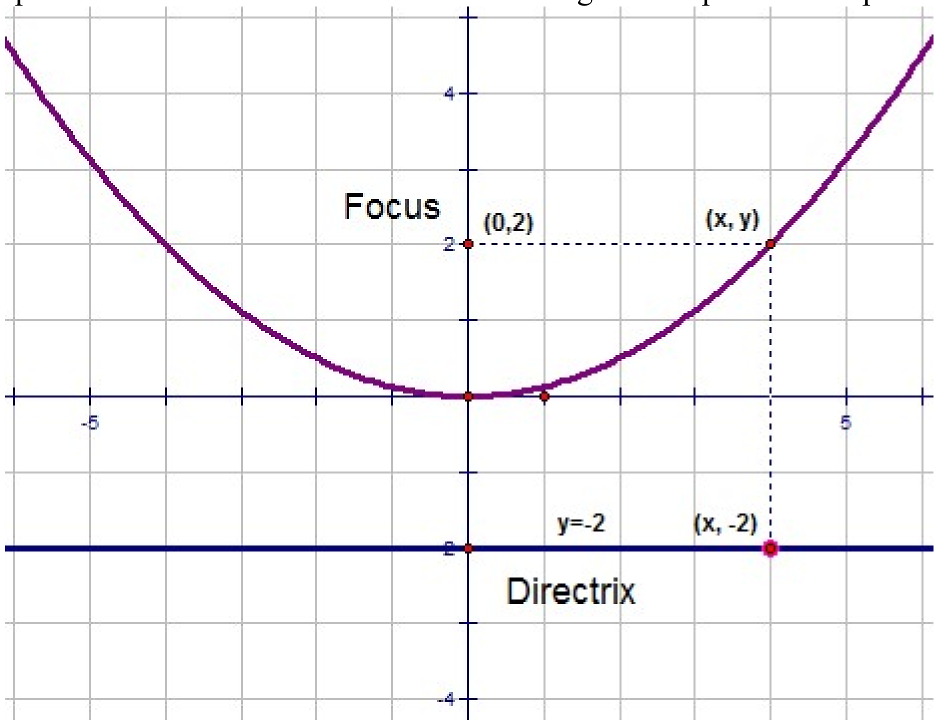
Part 1: Finding a specific equation.

Consider the parabola below. Notice the vertex at the origin and a point on the parabola (x, y).



Follow the steps to find the equation of the parabola.

1. Write equations for the distance from the focus to *(x, y)* and the distance from the directrix to *(x, y)*.

Solution:

Distance from point to focus: $d = \sqrt{(x-0)^2 + (y-2)^2}$

Distance from point to directrix: $d = \sqrt{(x-x)^2 + (y-(-2))^2}$

2. Because the definition of a parabola says that the distances you wrote in #1 are the same, write an equation stating this fact.

Solution:

$$\sqrt{(x-x)^2+(y-(-2))^2}=\sqrt{(x-0)^2+(y-2)^2}$$

3. Now square both sides to eliminate the radicals. Square the binomials and write your answer below.

Solution:

$$(x-x)^2+(y-(-2))^2=(x-0)^2+(y-2)^2$$
$$0^2+(y+2)^2=x^2+(y-2)^2$$
$$y^2+4y+4=x^2+y^2-4y+4$$

4. Collect like terms and solve the equation for *y*.

Solution:

$$y^2+4y+4=x^2+y^2-4y+4$$
$$8y=x^2$$
$$y=\frac{1}{8}x^2$$

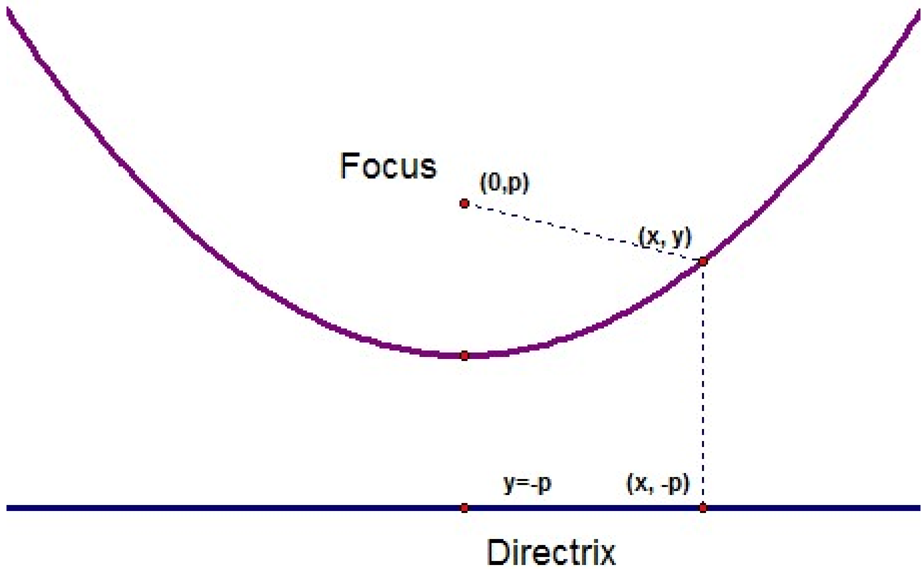
5. Write your equation in terms of *y* below.

Solution:
$$y=\frac{1}{8}x^2$$

Part 2: Writing the general equation for a Parabola.

Consider the graph of the parabola below. It's vertex is at the origin, the focus is at *(0,p)*, and the directrix is the line *y=-p*.

Follow the steps to find the equation of the parabola.



1. Write equations for the distance from the focus to *(x, y)* and the distance from the directrix to *(x, y)*.

Solution:

Distance from focus to point: $d = \sqrt{(x-0)^2 + (y-p)^2}$

Distance from directrix to point: $d = \sqrt{(x-x)^2 + (y-(-p))^2}$

2. Because the definition of a parabola says that the distances you wrote in #1 are the same, write an equation stating this fact.

Solution:

$$\sqrt{(x-x)^2 + (y-(-p))^2} = \sqrt{(x-0)^2 + (y-p)^2}$$

3. Now square both sides to eliminate the radicals. Square the binomials and write your answer below.

Comments:
The algebra gets a little tricky here for students, but they can do it! This is a part of the abstraction from definition to formula.

Solution:

$$(x-x)^2+(y-(-p))^2=(x-0)^2+(y-p)^2$$
$$0^2+(y+p)^2=x^2+(y-p)^2$$
$$y^2+2py+p^2=x^2+y^2-2py+p^2$$

4. Collect like terms and solve the equation for *y*.

Solution:

$$y^2+2py+p^2=x^2+y^2-2py+p^2$$
$$4py=x^2$$
$$y=\frac{1}{4p}x^2$$

5. Write your equation in terms of *y* below.

Solution:
$$y=\frac{1}{4p}x^2$$

6. What does the *"p"* represent in your equation?

Solution:
In the equation, p is always the distance from the focus to the vertex, or the distance from the vertex to the directrix.

Consider that the vertex of a parabola is not always at the origin. It could be translated to any point on the graph. Use what you know about transformations and your formula above to write the following equations. (all of the parabolas open either up or down)

Comment:
At this point, instead of symbolically manipulating the case with center (h, k), remind students about the transformations they have already learned. This will help develop the general formula for any parabola.

7. Vertex at *(2, 3)*; *p=4*. ***Solution:*** $y-3=\frac{1}{16}(x-2)^2$

8. Vertex at *(-4, 8); p=-3*. ***Solution:*** $y-8=-\frac{1}{12}(x+4)^2$

9. Vertex at *(5, -9); p=0.5*. ***Solution:*** $y+9=\frac{1}{2}(x-5)^2$

10. Vertex at *(h, k); p=p*. ***Solution:*** $y-k=\frac{1}{4p}(x-h)^2$