PROVING THE TANGENT ADDITION AND SUBTRACTION IDENTITIES

Georgia Standards of Excellence:

MGSE9-12.F.TF.9 Prove addition, subtraction, double and half-angle formulas for sine, cosine, and tangent and use them to solve problems.

Introduction:

This task uses algebraic manipulation and the previously developed identities to derive tangent addition and subtraction identities. Again, the emphasis should be on the mathematical processes and not the method itself. Students should feel confident in manipulating algebraic expressions and simplifying trigonometric expressions using identities.

PROVING THE TANGENT ADDITION AND SUBTRACTION IDENTITIES

By this point, you should have developed formulas for sine and cosine of sums and differences of angles. If so, you are already most of the way to finding a formula for the tangent of a sum of two angles.

Let's begin with a relationship that we already know to be true about tangent.

1. $\tan x = \dfrac{\sin x}{\cos x}$ so it stands to reason that $\tan\left(x+y\right) = \dfrac{\sin(x+y)}{\cos(x+y)}$

Use what you already know about sum and difference formulas to expand the relationship above.

2. $\tan\left(x+y\right) = \dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$

All Rights Reserved

3. Now we want to simplify this. (Hint: Multiply numerator and denominator by $\frac{1}{\cos x}$)

The reason to multiply by $\frac{1}{\cos x}$ is to establish a tangent ratio and to divide out cos x. It is important that students see why to choose that value as a part of the proof.

$$\tan(x+y) = \frac{\frac{\sin x \cos y}{\cos x} + \frac{\cos x \sin y}{\cos x}}{\frac{\cos x \cos y}{\cos x} - \frac{\sin x \sin y}{\cos x}} = \frac{\tan x \cos y + \sin y}{\cos y - \tan x \sin y}$$

4. You can simplify it some more. Think about step 3 for a hint.

Students should multiply by $\frac{1}{\cos y}$ to establish a tangent ratio for the y variable and divide out cos y. Lead them back to #3 if they need help.

$$\tan(x+y) = \frac{\frac{\tan x \cos y}{\cos y} + \frac{\sin y}{\cos y}}{\frac{\cos y}{\cos y} - \frac{\tan x \sin y}{\cos y}} = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

5. Write your formula here for the tangent of a sum:

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

6. Now that you have seen the process, develop a formula for the tangent of a difference.

Encourage students to attempt this on their own without referring back to the proof of the addition identity. If they need help, they may reference it. Encourage them to persevere through this process and not give up easily. Mathematics and especially proof is meant to be a productive struggle in which students work hard to construct their own knowledge.

When they have finished, they should have: $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

All Rights Reserved

Write your formula here:

$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y}$$

All Rights Reserved

THE TANGENT ADDITION AND SUBTRACTION IDENTITIES

By this point, you should have developed formulas for sine and cosine of sums and differences of angles. If so, you are already most of the way to finding a formula for the tangent of a sum of two angles.

Let's begin with a relationship that we already know to be true about tangent.

1. $\tan x = \dfrac{\sin x}{\cos x}$ so it stands to reason that $\tan\left(x+y\right) =$ ________________

Use what you already know about sum and difference formulas to expand the relationship above.

2. $\tan\left(x+y\right) =$

3. Now we want to simplify this. (Hint: Multiply numerator and denominator by $\dfrac{1}{\cos x}$)

4. You can simplify it some more. Think about step 3 for a hint.

5. Write your formula here for the tangent of a sum:

6. Now that you have seen the process, develop a formula for the tangent of a difference.

Write your formula here:

All Rights Reserved