CCGPS Precalculus
Unit 5 Matrices
WS -6 Determinants

Name: Key

Date: _____ Block: _____

EQ: How do we use determinants of matrices?

MM3A5: Students will perform basic operations with matrices.
MMA5.b: TLWBAT find the inverses of two-by-two matrices using pencil and paper, and find inverses of larger matrices using technology.

Evaluate the determinant of the matrix.

1. $\begin{bmatrix} -4 & 2 \\ 8 & 0 \end{bmatrix}$ $\det\begin{bmatrix} -4 & 2 \\ 8 & 0 \end{bmatrix} = -4(0) - 8(2) = \boxed{-16}$

2. $\begin{bmatrix} 1 & 4 \\ 5 & 1 \end{bmatrix}$ $\det\begin{bmatrix} 1 & 4 \\ 5 & 1 \end{bmatrix} = 1(1) - 4(5) = 1 - 20 = \boxed{-19}$

3. $A = \begin{bmatrix} -6 & 5 \\ 8 & 10 \end{bmatrix}$ $\det A = -6(10) - 8(5) = -60 - 40 = \boxed{-100}$

4. $B = \begin{bmatrix} 5 & 9 \\ 8 & 1 \end{bmatrix}$ $|B| = 5(1) - 9(8) = 5 - 72 = \boxed{-67}$

5. $\begin{bmatrix} 7 & -7 \\ 11 & 4 \end{bmatrix}$ $\begin{vmatrix} 7 & -7 \\ 11 & 4 \end{vmatrix} = 7(4) - 11(-7) = 28 + 77 = \boxed{105}$

6. $C = \begin{bmatrix} 1 & 3 \\ -2 & -6 \end{bmatrix}$ $|C| = 1(-6) - 3(-2) = -6 + 6 = \boxed{0}$

7. $D = \begin{bmatrix} 4 & 6 \\ 9 & 11 \end{bmatrix}$ $\det D = 11(4) - 9(6) = 44 - 54 = \boxed{-10}$

8. $E = \begin{bmatrix} 0 & 3 \\ -2 & 9 \end{bmatrix}$ $|E| = 0(9) - 3(-2) = \boxed{6}$

Evaluate the determinant of the matrix by using diagonals.

9. $A = \begin{bmatrix} 3 & 2 & -5 \\ 6 & 0 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ $\det A = [3(0)3 + 2(-1)0 + (-5)(6)(-1)] - [0(0)(-5) + (-1)(-1)(3) + 3(6)(2)) = [0 - 2 + 30] - [0 + 3 + 36] = \boxed{-11}$

10. $A = \begin{bmatrix} -1 & 2 & 7 \\ 2 & -1 & -1 \\ 3 & 5 & 2 \end{bmatrix}$ $\det A = (3 \cdot -1 \cdot 7 + 5 \cdot -1 \cdot -1 + 2 \cdot 2 \cdot 2) - (-1 \cdot -1 \cdot 2 + 2 \cdot -1 \cdot 3 + 7 \cdot 2 \cdot 5) = (-21 + 5 + 8) - (2 - 6 + 70) = \boxed{-74}$

11. $A = \begin{bmatrix} 1 & -2 & 1 \\ 6 & 5 & 0 \\ 1 & 4 & -2 \end{bmatrix}$ $\det A = [1 \cdot 5 \cdot -2 + 2 \cdot 0 \cdot 1 + 1 \cdot 6 \cdot 4] - [1 \cdot 5 \cdot 1 + 4 \cdot 0 \cdot 1 + -2 \cdot 6 \cdot 2] = [-10 + 0 + 24] - [5 + 0 - 24] = [14 + 19] = \boxed{33}$

12. $A = \begin{bmatrix} 3 & 12 & 1 \\ -10 & 9 & 8 \\ -5 & 4 & -1 \end{bmatrix}$ $\det A = \boxed{-718}$

Solve for x.

13. $\begin{vmatrix} 2 & 6 \\ 1 & x \end{vmatrix} = 2$

$2x - 6 = 2$

$2x = 8$

$\boxed{x = 4}$

14. $\begin{vmatrix} x & 3 & -1 \\ 2 & 1 & -2 \\ 4 & 1 & x \end{vmatrix} = 10$

$(x \cdot 1 \cdot x + 3 \cdot -2 \cdot 4 + -1 \cdot 2 \cdot 1) - (4 \cdot 1 \cdot -1 + 1 \cdot -2 \cdot x + x \cdot 2 \cdot 3) = 10$

$x^2 - 24 - 2 - (-4 - 2x + 6x) = 10$

$x^2 - 26 + 4 - 4x = 10$

$x^2 - 4x - 22 = 10$

$x^2 - 4x - 32 = 0$

$(x - 8)(x + 4) = 0$

$\boxed{x = 8, -4}$

15. $\begin{vmatrix} x & 3 \\ -4 & x \end{vmatrix} = 7x$

$x^2 + 12 = 7x$

$x^2 - 7x + 12 = 0$

$(x-3)(x-4) = 0$

$x = 3, 4$

16. $\begin{vmatrix} 2x & 0 & 3 \\ 7 & 5 & -1 \\ 4 & 2 & x \end{vmatrix} = 9x^2 - 3x + 12$

$(2x \cdot 5 \cdot x + 0 \cdot -1 \cdot 4 + 3 \cdot 7 \cdot 2) - (4 \cdot 5 \cdot 3 + 2 \cdot -1 \cdot 2x + x \cdot 7 \cdot 0) = 9x^2 - 3x + 12$

$10x^2 + 42 - (60 - 4x) = 9x^2 - 3x + 12 \Rightarrow x^2 + 7x - 30 = 0$

$-60 + 4x$

$(x-3)(x+10) = 0$

$x = 3, -10$

17. Given a triangle with the following vertices, find the area using the determinant formula.

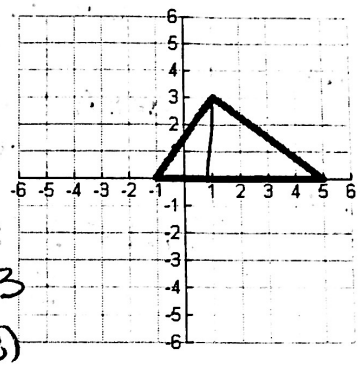
a. (-1, 6), (4, 7), and (8, -6)

$A = \pm\frac{1}{2}$ $\begin{vmatrix} -1 & 6 & 1 \\ 4 & 7 & 1 \\ 8 & -6 & 1 \end{vmatrix} = \pm\frac{1}{2}(-69) = \frac{69}{2}$

b. (-1, -3), (4, 2), and (7, 3)

$A = \pm\frac{1}{2}$ $\begin{vmatrix} -1 & -3 & 1 \\ 4 & 2 & 1 \\ 7 & 3 & 1 \end{vmatrix} = 5$

18. Given a triangle with vertices (-1, 0), (1, 3), and (5, 0), find the area using the determinant formula. Verify that area you found is correct using geometric formulas.



$A = \pm\frac{1}{2}$ $\begin{vmatrix} -1 & 0 & 1 \\ 1 & 3 & 1 \\ 5 & 0 & 1 \end{vmatrix} = 9$

$A = \frac{1}{2}b \cdot h$

$= \frac{1}{2} 6 \cdot 3$

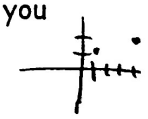
$= \frac{1}{2}(18)$

$= 9$

19. Suppose you are finding the area of a triangle with vertices (-1, -1), (4, 7), and (9, -6). You find the area of the triangle to be -52.5 and your partner works the same problem and gets +52.5. After checking both solutions, you each have done your work correctly. How can you explain this discrepancy?

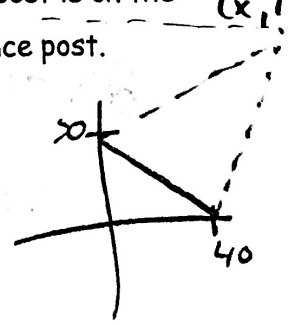
The order for which you put the coordinates into the matrices was different.

20. Suppose another triangle with vertices (1, 1), (4, 2), and (7, 3) gives an area of 0. What do you know about the triangle and the points?



All the points are on the same line so no Δ is formed.

21. A gardener is trying to find a triangular area behind his house that encloses 1750 square feet. He has placed the first two fence posts at (0, 50) and at (40, 0). The final fence post is on the property line at y = 100. Find the point where the gardener can place the final fence post.



$\frac{1}{2}$ $\begin{vmatrix} 0 & 50 & 1 \\ 40 & 0 & 1 \\ x & 100 & 1 \end{vmatrix} = 1750$

$0 + 50x + 4000 - 2000 = 3500$

$50x + 2000 = 3500$

$(30, 100)$

$x = 30$