WS Trig Identities

Remind students #1-4, must use Pythag Trig Ident, not Δ's

1) $\sin x = \frac{\sqrt{3}}{2}$ $\cos x = \frac{1}{2}$

$\csc = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ $\sec x = 2$

$1 + \tan^2 x = \sec^2 x$ $\cot^2 x + 1 = \csc^2 x$

$\tan = \sqrt{\sec^2 x - 1}$ $\cot x = \sqrt{\csc^2 x - 1} = \sqrt{\left(\frac{2\sqrt{3}}{3}\right)^2 - 1}$

$= \sqrt{2^2 - 1} = \sqrt{3}$ $= \sqrt{\frac{12}{9} - 1} = \sqrt{\frac{1}{3}} = \frac{\sqrt{3}}{3}$

$\csc x = 2\sqrt{3}/3$

$\sec x = 2$

$\tan x = \sqrt{3}$

$\cot x = \sqrt{3}/3$

2) $\tan x = \frac{7}{24}$ $\sec x = -\frac{25}{24}$ Q3

$\cot x = 24/7$ $\cos x = -24/25$

$\csc x = \sqrt{\cot^2 x + 1}$ $\sin x = \sqrt{1 - \cos^2 x}$

$= \sqrt{\left(\frac{24}{7}\right)^2 + 1}$ $= \sqrt{1 - \left(\frac{24}{25}\right)^2} = \sqrt{\frac{49}{625}}$

$= \sqrt{\frac{576}{49} + \frac{49}{49}} = \sqrt{\frac{625}{49}} = \frac{25}{7}$ $= \frac{7}{25}$ ← Q3, must be neg

$\sin(-x) = -\sin x$

$\sin x = -7/25$

$\cos x = -24/25$

$\csc x = -25/7$

$\cot x = 24/7$

3) $\sin(-x) = \frac{-2}{3}$ $\tan x = \frac{-2\sqrt{5}}{5}$ Quad 2

$\sin(x) = 2/3$ $\cot x = \frac{-5}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{-5\sqrt{5}}{10} = \frac{-\sqrt{5}}{2}$

$\cos x = \sqrt{1 - \sin^2 x}$ $\csc x = \frac{1}{\sin x} = \frac{3}{2}$

$= \sqrt{1 - \left(\frac{2}{3}\right)^2} = \pm\sqrt{\frac{5}{9}} = \frac{-\sqrt{5}}{3}$

$\sec x = \frac{-3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{-3\sqrt{5}}{5}$

$\csc x = 3/2$

$\sin x = 2/3$

$\cos x = -\sqrt{5}/3$

$\csc x = 3/2$

$\sec x = -3\sqrt{5}/5$

$\cot x = -\sqrt{5}/2$

4) $\sin\theta = -1$ $\cot\theta = 0$

$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-1} = -1$ $\tan\theta = \frac{1}{\cot 0} = \frac{1}{0} = \text{und}$

$\cos^2 + \sin^2 = 1$ $\sec\theta = \frac{1}{\cos 0} = \frac{1}{0} = \text{und}$

$\cos = \sqrt{1 - \sin 2} = \sqrt{1 - (-1)^2}$

$= 0$

$\cos\theta = 0$

$\tan\theta = \text{und}$

$\csc\theta = -1$

$\sec\theta = \text{und}$

5) $\csc x \cdot \sin x = \frac{1}{\sin x} \cdot \frac{\sin x}{1} = 1$ d

6) $\tan x \cdot \cos x = \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} = \sin x$ f

7) $\tan^2 x - \sec^2 x = \frac{\sin^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} = \frac{-\cos^2 x}{\cos^2 x} = -1$ a

8) $(1 - \sin^2 x)(\sec x) = \cos^2 x \cdot \frac{1}{\cos x} = \cos x$ b

9) $\frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{+\cos x} = -\tan x$ e

10) $\frac{\sin\left[\frac{\pi}{2} - x\right]}{\cos\left[\frac{\pi}{2} - x\right]} = \frac{\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x}{\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x} = \frac{(1)\cos x - 0 \cdot \sin x}{0 \cdot \cos x + 1 \cdot \sin x} = \frac{\cos x}{\sin x} = \cot x$ c

11) $\cos x \cdot \csc x$

$\frac{\cos x}{1} \cdot \frac{1}{\sin x}$

$= \cot x$ {b}

12) $\sin^2 x (\csc^2 x - 1)$

$\sin^2 x \left(\frac{1}{\sin^2 x} - 1\right)$

$1 - \sin^2 x$

$= \cos^2 x$ {c}

13) $\sec^4 x - \tan^4 x$

$(\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)$

$(\sec^2 x + \tan^2 x)(\tan^2 x + 1 - \tan^2 x)$

$(\sec^2 x + \tan^2 x)(1)$ {f}

14) $\cot x \sec x$

$\frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}$

$\frac{1}{\sin x} = \csc x$ {a}

15) $\frac{\sec^2 x - 1}{\sin^2 x} = \frac{\tan^2 x + 1 - 1}{\sin^2 x}$

$= \frac{\sin^2 x}{\cos^2 x} \cdot \frac{1}{\sin^2 x} =$

$\frac{1}{\cos^2 x} = \sec^2 x$ {e}

16) $\frac{\cos^2\left[\frac{\pi}{2} - x\right]}{\cos x}$

$= \frac{\cos^2\left(\frac{\pi}{2}\right)\cos^2 x + \sin^2\left(\frac{\pi}{2}\right)\sin^2 x}{\cos x}$

$= \frac{0 \cdot \cos^2 x + 1 \cdot \sin^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$

$= \frac{\sin x \cdot \sin x}{\cos x} = \tan x \sin x$ {d}

17) $\cot^2 x - \cot^2 x \cos^2 x$

$\cot^2 x (1 - \cos^2 x)$

$\frac{\cos^2 x}{\sin^2 x} \cdot \sin^2 x = \cos^2 x$

18) $\sin^2 x \cdot \sec^2 x - \sin^2 x$

$\sin^2 x (\sec^2 x - 1)$

$\tan^2 x + 1 = \sec^2 x$

$\sin^2 x (\tan^2 x + 1 - 1)$

$\sin^2 x \tan^2 x$

19) $\tan^4 x + 2\tan^2 x + 1$ {Perfect Sq Trin

$(\tan^2 x + 1)^2$

$(\sec^2 x)^2$

$\sec^4 x$

DOTS

20) $\sin^4 x - \cos^4 x$

$(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)$

$1 (\sin^2 x - \cos^2 x)$

$-\cos 2x$

21) $(\sin x + \cos x)^2$

$\sin^2 x + 2\sin x \cos x + \cos^2 x$

$1 + 2\sin x \cos x$

$= 1 + \sin 2x$

22) $(\sec x + 1)(\sec x - 1)$

$\sec^2 x - 1$

$\tan^2 x + 1 - 1$

$\tan^2 x$

WS Trig Identities

23) $\frac{1}{1+\cos X} + \frac{1}{1-\cos X}$

$\frac{(1-\cos X)}{(1-\cos X)} \quad \frac{(1+\cos X)}{(1+\cos X)}$

$\frac{1-\cos X + 1+\cos X}{1-\cos^2 X}$

$\frac{2}{\sin^2 X} = 2\csc^2 X$

24) $\frac{\cos X}{1+\sin X} + \frac{1+\sin X}{\cos X}$

$\cdot \frac{\cos X}{\cos X} \quad \cdot \frac{1+\sin X}{1+\sin X}$

$= \frac{\cos^2 X + 1 + 2\sin X + \sin^2 X}{\cos X(1+\sin X)}$

$= \frac{1+1+2\sin X}{\cos X(1+\sin X)}$ $= 2+2\sin X$

$= \frac{2(1+\sin X)}{\cos X(1+\sin X)}$

$= 2\sec X$

2 sec X

25) $\frac{\sin^2 y}{1-\cos y} \cdot \frac{1+\cos y}{1+\cos y}$

$= \frac{\sin^2 y(1+\cos y)}{1-\cos^2 y}$

$= \frac{\sin^2 y(1+\cos y)}{1-(1-\sin^2 y)}$

$= \frac{\sin^2 y(1+\cos y)}{\sin^2 y}$

$= 1+\cos y$

26) $\frac{\csc\theta}{\sec\theta} + \frac{\cos\theta}{\sin\theta} = 2\cot\theta$

$\frac{\cos\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = 2\cot\theta$

$\cot\theta + \cot\theta = 2\cot\theta$

$2\cot\theta = 2\cot\theta$

27) $1 - \frac{\sin^2\theta}{1-\cos\theta} = -\cos\theta$

$\frac{1+\cos\theta}{1+\cos\theta}\left(\quad\right) =$

$= 1 - \frac{(1+\cos\theta)\sin^2\theta}{1-\cos^2\theta} = -\cos\theta$

$= 1 - \frac{(1+\cos\theta)(\sin^2\theta)}{\sin^2\theta} = -\cos\theta$

$= 1 - 1 - \cos\theta = -\cos\theta$

$-\cos\theta = -\cos\theta$

28) $\frac{\cot(-\theta)}{\csc\theta} = -\cos\theta$

$\frac{\cos(-\theta)}{\sin(-\theta)} \cdot \sin\theta = -\cos\theta$

$\frac{\cos\theta}{-\sin\theta} \cdot \sin\theta = -\cos\theta$

$-\cos\theta = -\cos\theta$

29) $\sin\theta + \cos\theta\cot\theta = \csc\theta$

$\sin\theta + \frac{\cos\theta}{1} \cdot \frac{\cos\theta}{\sin\theta} =$

$\sin\theta + \frac{\cos^2\theta}{\sin\theta} =$

$\frac{\sin\theta}{\sin\theta}\left(\quad\right)$

$\frac{\sin^2\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin\theta} =$

$\frac{1}{\sin\theta}$

$\csc\theta = \csc\theta$

30) $\frac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta$

$= \frac{1+\sin\theta}{1+\sin\theta}\left(\frac{\cos\theta}{1-\sin\theta}\right) = \sec\theta + \tan\theta$

$= \frac{\cos\theta + \sin\theta\cos\theta}{1-\sin^2\theta} =$

$= \frac{\cos\theta + \sin\theta\cos\theta}{\cos^2\theta}$

$= \underbrace{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$

$\sec\theta + \tan\theta = \sec\theta + \tan\theta$

31) $\frac{\sin\theta}{\csc\theta} + \frac{\cos\theta}{\sec\theta} = 1$

$\sin\theta \cdot \sin\theta + \cos\theta\cos\theta = 1$

$\sin^2\theta + \cos^2\theta = 1$

$1 = 1$

32) $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\csc\theta$

$\frac{\sin\theta}{\sin\theta}\left(\quad\right) + \frac{1-\cos\theta}{1-\cos\theta}\left(\quad\right) = 2\csc\theta$

$\frac{\sin\theta + \sin\theta\cos\theta + \sin\theta - \sin\theta\cos\theta}{\sin^2\theta} =$

$\frac{2\sin\theta}{\sin^2\theta} =$

$\frac{2}{\sin\theta}$

$2\csc\theta = 2\csc\theta$