WS 3: Sum & Diff Formulas

Part I

1) $\cos 40^\circ \cos 15^\circ - \sin 40^\circ \sin 15$

$\cos(40+15)^\circ$

$\cos 55^\circ$

$= 0.5736$ ☺

2) $\dfrac{\tan 325 - \tan 86}{1 + \tan 325 \tan 86}$

$= \tan(325 - 86)$

$= \tan(239)$

$= 1.66$ ☺

3) $\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{\pi}{5}\right) - \sin\left(\frac{\pi}{7}\right)\sin\left(\frac{\pi}{5}\right)$

$\cos\left(\frac{\pi}{7} + \frac{\pi}{5}\right)$

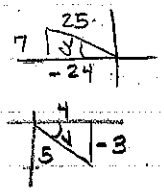
$\cos\left(\frac{5\pi}{35} + \frac{7\pi}{35}\right)$

$\cos\left(\frac{12\pi}{35}\right)$

0.9998 ☺

Part II

$\sin u = \frac{7}{25}$ $\quad \pi/2 < u < \pi$



$\therefore u = 163.739$

$\cos v = \frac{4}{5}$ $\quad \frac{3\pi}{2} < v < 2\pi$

$v = +323.130$

4. $\cos(u+v)$

$= \cos(u)\cos(v) - \sin(u)\sin(v)$

$= \left(-\frac{24}{25}\right)\left(\frac{4}{5}\right) - \left(\frac{7}{25}\right)\left(\frac{-3}{5}\right)$

$= -96/125 + 21/125$

$= -75/125$

$= -3/5$

5) $\sin(v-u)$

$= \sin v \cos u - \cos v \sin u$

$= \left(\frac{-3}{5}\right)\left(\frac{-24}{25}\right) - \left(\frac{4}{5}\right)\left(\frac{7}{25}\right)$

$= 72/125 - 28/125$

$= 44/125$

Part III

6) $\cos(\pi - \theta) + \sin\left(\frac{\pi}{2} + \theta\right) = 0$

$\cos\pi\cos\theta + \sin(\pi)\sin(\theta) +$

$\sin\left(\frac{\pi}{2}\right)\cos(\theta) + \cos\left(\frac{\pi}{2}\right)\sin\theta = 0$

$(-1)\cos\theta + 0\sin\theta + 1\cdot\cos\theta + 0\cdot\sin\theta = 0$

$-\cos\theta + 0 + \cos\theta + 0 = 0$

$0 = 0$

7) $\tan(x+\pi) - \tan(\pi - x) = 2\tan x$

$\dfrac{\tan x + \tan\pi}{1 - \tan x \tan\pi} - \left[\dfrac{\tan\pi - \tan x}{1 + \tan\pi \tan x}\right]$

$\dfrac{\tan x + 0}{1 - \tan x \cdot 0} - \left[\dfrac{0 - \tan x}{1 + 0\cdot\tan x}\right]$

$\tan x - (-\tan x)$

$2\tan x = 2\tan x$

8) $\sin(x+y) + \sin(x-y) = 2\sin x \cos y$

$\sin x\cos y + \cos x\sin y + \sin x\cos y - \cos x\sin y =$

(cancel)

$2\sin(x)\cos(y) = 2\sin(x)\cos(y)$

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Part II $[0, 2\pi)$

9. $\sin\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)=1$

$\sin(x)\cos\left(\frac{\pi}{3}\right)+\cos(x)\sin\left(\frac{\pi}{3}\right)$

$+\sin(x)\cos\left(\frac{\pi}{3}\right)-\cos(x)\sin\left(\frac{\pi}{3}\right)$

$\sin(x)\cdot\frac{1}{2}+\cos(x)\cdot\frac{\sqrt{3}}{2}+\sin(x)\cdot\frac{1}{2}-\cos(x)\cdot\frac{\sqrt{3}}{2}$

cancel

$\frac{1}{2}\sin(x)+\frac{1}{2}\sin(x)=1$

$\sin x=1 \quad x=\sin^{-1}(1)$

$x=\pi/2$

$\frac{\pi}{2}+\frac{\pi}{3}$

$\frac{3\pi}{6}+\frac{2\pi}{6}$

$5\pi/6 \qquad \pi/6$

10. $\cos\left(x+\pi/4\right)-\cos\left(x-\pi/4\right)=1$

$\cos(x)\cos(\pi/4)-\sin(x)\sin(\pi/4)$

$-\left[\cos(x)\cos(\pi/4)+\sin(x)\sin(\pi/4)\right]=1$

$\frac{\sqrt{2}}{2}\cos(x)-\frac{\sqrt{2}}{2}\sin(x)-\left[\frac{\sqrt{2}}{2}\cos(x)+\frac{\sqrt{2}}{2}\sin(x)\right]=1$

cancel

$-\frac{\sqrt{2}}{2}\sin(x)-\frac{\sqrt{2}}{2}\sin x=1$

$-\sqrt{2}\sin x=1$

$\sin x=\frac{1}{-\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$

$x=5\pi/4,\ 7\pi/4$ $\qquad x=3.927,\ 5.498$

V: $\cos\left(\frac{5\pi}{4}+\frac{\pi}{4}\right)-\cos\left(\frac{5\pi}{4}-\frac{\pi}{4}\right)=1$

$\cos\left(\frac{6\pi}{4}\right)-\cos\left(\frac{4\pi}{4}\right)$

$\cos\left(\frac{3\pi}{4}\right)-\cos\pi=1$

$0-(-1)=1$

☺ $\quad 1=1$