$x^2 - 1$
$(x+1)(x-1)$
$(\cos x + 1)(\cos x - 1) = 0$

Solving Trig Equations Part 2

Quadratic Trig Equations - *Reminder, $\sin^2 x = 1 - \cos^2 x$
Solve $0 \le x < 2\pi$:

$\sin^2 x - \frac{1}{4} = 0$
$\sin^2 x = \frac{1}{4}$
$\sin x = \pm\frac{1}{2}$
$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

$\cos^2 x - 1 = 0$
$\cos^2 x = 1$
$\cos x = \pm 1$
$\boxed{x = 0, \pi}$

Last day we solved linear equations, what about factored ones $0 \le x < 2\pi$?

$(x-5)(x+3) = 0$

$(\cos x - 1)(\tan x - 1) = 0$
$\cos x - 1 = 0 \qquad \tan x - 1 = 0$
$\cos x = 1 \qquad \tan x = 1$
$x = 0 \qquad x = \frac{\pi}{4}, \frac{5\pi}{4}$
$\boxed{x = 0, \frac{\pi}{4}, \frac{5\pi}{4}}$

$2\cos x\left(\cos x + \frac{1}{2}\right) = 0$
$2\cos x = 0 \qquad \cos x + \frac{1}{2} = 0$
$\cos x = 0 \qquad \cos x = -\frac{1}{2}$
$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \frac{2\pi}{3}, \frac{4\pi}{3}$
$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}}$

So far our newest strategy is to factor equations in order to solve them:
One thing that can help is making a substitution

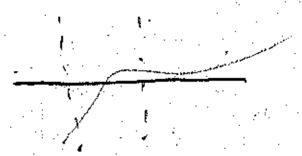
Trig expression	*Substitution*
$\sin^2 x - 3\sin x = \sin x(\sin x - 3)$	$b^2 - 3b = b(b-3)$
$\csc^2 x - 3\csc x - 28 = (\csc x - 7)(\csc x + 4)$	$b^2 - 3b - 28 = (b-7)(b+4)$
$2\cos^2 x + 7\cos x - 4 = (2\cos x - 1)(\cos x + 4)$	$2b^2 + 7b - 4 = 2b^2 + 8b - 1b - 4$ $2b(b+4) - 1(b+4)$ $= (2b-1)(b+4)$

Solve by factoring $0 \le x < 2\pi$:

$2\sin^2 x - \sin x = 0$
$\sin x(2\sin x - 1) = 0$
$\sin x = 0 \qquad 2\sin x - 1 = 0$
$x = 0, \pi \qquad \sin x = \frac{1}{2}$
$x = \frac{\pi}{6}, \frac{5\pi}{6}$
$\boxed{x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}}$

$\sin x \cos x = 2\cos x$
$\sin x \cos x - 2\cos x = 0$
$\cos x(\sin x - 2) = 0$
$\cos x = 0 \qquad \sin x - 2 = 0$
$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}} \qquad \sin x \ne 2$

*Be careful with the tangent ratio!
$\tan x \sin x + \tan x = 0$
$\tan x(\sin x + 1) = 0$
$\tan x = 0 \qquad \sin x + 1 = 0$
$x = 0, \pi \qquad \sin x = -1$
$x = \frac{3\pi}{2}$
$\boxed{x = 0, \pi, \frac{3\pi}{2}}$



Solve by factoring and state the general solution:

$\cos^2 x - \cos x - 2 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$(\cos x - 2)(\cos x + 1) = 0$

$\cos x - 2 = 0 \quad \cos x + 1 = 0$

$\cos x \neq 2 \quad \cos x = -1$

$\boxed{x = \pi + 2\pi k}$

$2\sin^2 x - 3\sin x + 1 = 0$

$2x^2 - 3x + 1 = 0$

$(2x-1)(x-1)$

$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x - 1 = 0 \quad \sin x - 1 = 0$

$\sin x = \frac{1}{2} \quad \sin x = 1$

$$\boxed{\begin{array}{l} x = \frac{\pi}{6} + 2\pi k \\ \frac{5\pi}{6} + 2\pi k \\ \frac{\pi}{2} + 2\pi k \end{array}}$$

Some equations are very difficult to solve algebraically:

There are two methods for solving graphically that are commonly used,

1) method of intersection 2) intercept method

I prefer the intercept method because it is easier to set up your window

1) Set up the equation
2) 2nd Trace (calc)
3) 2: zeros
4) window
   - x → should match the domain
   - y → can be -1 to 1 (anything that shows the x-axis)

Ex.

Solve graphically $0 \le x < 2\pi$:

$\cos^3 x - 3\cos x + 1 = 0$

$x = 1.22, 5.07$

$\sin x = \frac{1}{x}$

$x = 2.77, .90$
$(.8975)$

Solving graphically with exact values:

It is possible to solve exactly with the graphing calculator. The adjustment we have to make is on the x-scale:

Let's look at the special angles:

$0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \ldots$ What is the LCD of the special angles?

If we set the x-scale to the LCD, then we can see which 'tick' mark the graph crosses at.

Ex.

Solve exactly $0 \le x < 2\pi$

$2\sin^2 x = 1 - \sin x$

$x = \frac{2\pi}{12} = \frac{\pi}{6}$

$\frac{10\pi}{12} = \frac{5\pi}{6}$

$\frac{18\pi}{12} = \frac{3\pi}{2}$