

# WS1 Solving Trig Eqns

## Part I

1)  $2\cos x + 1 = 0$   
 $\cos x = -1/2$

$x = \arccos(-1/2)$

a)  $x = \frac{2\pi}{3}, \frac{4\pi}{3} \quad [0, 2\pi)$

and

b)  $x = \frac{2\pi}{3} + 2\pi k$   
 $x = \frac{4\pi}{3} + 2\pi k \quad \left. \right\} (-\infty, \infty)$

2)  $3\csc^2 x - 4 = 0$

$\csc^2 x = 4/3$

$\frac{1}{\sin^2 x} = 4/3$

$\sin^2 x = 3/4$

$\sin x = \pm \sqrt{3}/2$

a)  $x = \frac{\pi}{3}, \frac{2\pi}{3} \quad [0, 2\pi)$   
 $x = \frac{4\pi}{3}, \frac{5\pi}{3}$

b)  $x = \frac{\pi}{3} + \pi k$   
 $x = \frac{2\pi}{3} + \pi k \quad \left. \right\} (-\infty, \infty)$

3)  $4\cos^2 x - 3 = 0$

$\cos^2 x = 3/4$

$\cos x = \pm \sqrt{3}/2$

a)  $x = \frac{\pi}{6}, \frac{11\pi}{6} \quad [0, 2\pi)$   
 $x = \frac{5\pi}{6}, \frac{7\pi}{6}$

b)  $x = \frac{\pi}{6} + \pi k$   
 $x = \frac{5\pi}{6} + \pi k \quad \left. \right\} (-\infty, \infty)$

4)  $(3\tan^2 x - 1)(\tan^2 x - 3) = 0$

$3\tan^2 x - 1 = 0$

$\tan^2 x = 1/3$

$\tan x = \pm \frac{1}{\sqrt{3}}$

$\frac{\sin x}{\cos x} = \pm \frac{1/\sqrt{3}}{1/2}$

$\tan^2 x = \frac{3}{1}$

$\tan x = \pm \sqrt{3}$

$\frac{\sin x}{\cos x} = \pm \frac{\sqrt{3}/2}{1/2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

$x = \frac{4\pi}{3}, \frac{5\pi}{3}$

a)  $x = \frac{\pi}{6}, \frac{11\pi}{6}$   
 $x = \frac{5\pi}{6}, \frac{7\pi}{6}$

$[0, 2\pi)$

b)  $x = \frac{\pi}{6} + \pi k$

$x = \frac{5\pi}{6} + \pi k$

$x = \frac{\pi}{3} + \pi k$

$x = \frac{2\pi}{3} + \pi k$

$(-\infty, \infty)$

or  
 $\frac{\pi}{6} + \frac{\pi}{2}k$   
 $\frac{\pi}{3} + \frac{\pi}{2}k$

## Part II

5)  $3\tan^3 x = \tan x$

$3\tan^3 x - \tan x = 0$

$(\tan x)(3\tan^2 x - 1) = 0$

$\tan x = 0 \quad 3\tan^2 x - 1 = 0$

$\tan x = \pm \sqrt{\frac{1}{3}} = \pm \frac{1/\sqrt{3}}{1/2}$

a)  $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \quad [0, 2\pi)$   
 $x = 0, \pi$

b)  $x = 0 + \pi k$   
 $x = \frac{\pi}{6} + \pi k$   
 $x = \frac{5\pi}{6} + \pi k \quad \left. \right\} (-\infty, \infty)$

6)  $2\sin x + \csc x = 0$

$\left[ 2\sin x + \frac{1}{\sin x} = 0 \right] \sin x$

$2\sin^2 x + 1 = 0$

$\sin^2 x = -1/2$

$\sin x = \pm \sqrt{-1/2}$  Fails

No Solutions

# WS-7: Solving Trig Equations

7)  $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{2}}{2}$   
 $\frac{1}{2}x = \frac{\pi}{4} \quad \frac{1}{2}x = \frac{7\pi}{4}$   
 $x = \frac{2\pi}{4} \quad x = \frac{14\pi}{4} = 3\frac{1}{2}\pi$  Fail, too big  
 a)  $x = \frac{\pi}{2} \quad [0, 2\pi)$

b)  $\frac{1}{2}x = \frac{\pi}{4} + 2\pi k \quad \frac{1}{2}x = \frac{7\pi}{4} + 2\pi k$   
 $x = \frac{\pi}{2} + 4\pi k \quad x = \frac{7\pi}{2} + 4\pi k$   
 $(-\infty, \infty)$

8)  $2\sec^2 x + \tan^2 x - 3 = 0$   
 $2(\tan^2 x + 1) + \tan^2 x - 3 = 0$   
 $2\tan^2 x + 2 + \tan^2 x - 3 = 0$   
 $3\tan^2 x - 1 = 0$   
 $\tan^2 x = \frac{1}{3}$   
 $\tan x = \pm \frac{1}{\sqrt{3}}$

$\frac{\sin x}{\cos x} = \pm \frac{1/2}{\sqrt{3}/2}$   
 a)  $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \quad [0, 2\pi)$

b)  $x = \frac{\pi}{6} + \pi k$   
 $x = \frac{5\pi}{6} + \pi k \quad (-\infty, \infty)$

9)  $\sin 2x = -\frac{\sqrt{3}}{2}$   
 $2x = \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$   
 $x = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

a)  $x = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6} \quad [0, 2\pi)$

b)  $x = \frac{2\pi}{3} + \pi k$   
 $x = \frac{5\pi}{6} + \pi k \quad (-\infty, \infty)$

10)  $\tan 3x = 1$   
 $3x = \frac{\pi}{4} \quad 3x = \frac{5\pi}{4}$   
 a)  $x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{21\pi}{12}$   
 $[0, 2\pi)$

b)  $x = \frac{\pi}{12} + \frac{1}{3}\pi k \quad (-\infty, \infty)$

11)  $9\cos x + 2 = 3$  OR Graphing  
 $9\cos x = 1$   
 $\cos x = \frac{1}{9}$   
 $x = \arccos \frac{1}{9}$   
 $x = 1.459, 4.824$   
 $0 = 9\cos x - 1$

12)  $4\sin^3 x + 2\sin^2 x = 2\sin x + 1$   
 $4\sin^3 x + 2\sin^2 x - 2\sin x - 1 = 0$   
 $2\sin^2 x \left( \frac{2\sin x + 1}{2\sin x + 1} - 1 \right) - 1 = 0$

$(2\sin^2 x - 1)(2\sin x + 1) = 0$   
 $\rightarrow \sin x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$   $\left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$   
 $x = 0.7854, 2.3562, 3.9270, 5.4978$

and  $2\sin x + 1 = 0$   
 $\sin x = -\frac{1}{2}$   
 $x = 5.7596, 7.7596 \quad \left\{ \frac{7\pi}{6}, \frac{11\pi}{6} \right\} \quad (-0.5236)$   
 $3.6652 \quad \frac{7\pi}{6}$

