**CCGPS Precalculus** **Name** _____

**Unit 5: Matrices Review Sheet**

## NON-CALCULATOR SECTION

**Find the determinant.**

1. $\begin{bmatrix} 2 & 1 & 1 \\ 7 & 4 & -3 \\ -1 & 5 & 1 \end{bmatrix} = 73$

2. $\begin{bmatrix} 10 & 4 & 6 \\ 2 & -3 & 1 \\ -3 & 2 & 0 \end{bmatrix} = -62$

3. **Multiply if possible.** $\begin{bmatrix} -1 & 4 & -2 \\ 1 & -1 & -4 \end{bmatrix} \cdot \begin{bmatrix} -6 & 2 & -5 \\ 1 & -4 & 2 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -18 & 15 \\ -19 & 6 & -3 \end{bmatrix}$

**Find the inverse, if it exists.**

4. $\begin{bmatrix} 21 & 12 \\ 7 & 4 \end{bmatrix}$ det A = 0, no inverse exists

5. **Find the area of a triangle with vertices (3, -2), (-1, -7), (4, 11)** 23.5 units$^2$

**Name the dimensions of the matrices.**

6. $\begin{bmatrix} 3 & 2 & 1 \\ -5 & 6 & -3 \end{bmatrix}$ 2x3

7. $\begin{bmatrix} 8 \\ 9 \\ -2 \\ 3 \end{bmatrix}$ 4x1

8. $\begin{bmatrix} 2 & 43 \end{bmatrix}$ 1x2

**Perform indicated operations. #9-13**

9. $-5\begin{bmatrix} 6 & 2 & 0 \\ 4 & -11 & 6 \end{bmatrix} = \begin{bmatrix} -30 & -10 & 0 \\ -26 & 55 & -30 \end{bmatrix}$

10. $4\begin{bmatrix} -8 & 10 \\ 0 & -4 \end{bmatrix} - 5\begin{bmatrix} -1 & 4 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} -27 & 20 \\ -20 & -51 \end{bmatrix}$

11. $\begin{bmatrix} -6 & 9 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 5 & -2 & 4 \\ 1 & 0 & -5 \end{bmatrix} = \begin{bmatrix} -21 & 12 & -69 \\ 9 & -4 & 13 \end{bmatrix}$

12. $\begin{bmatrix} 7 & 0 & -8 \\ 10 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ 4 & 0 \end{bmatrix}$ Undefined

13. $8\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} + 3\begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix} - \begin{bmatrix} -1 \\ 8 \\ -6 \end{bmatrix} = \begin{bmatrix} 29 \\ -34 \\ 59 \end{bmatrix}$

**Solve for x.**

**Hint: There are bars, not brackets, around the matrices.**

14. $\begin{vmatrix} 5 & -4 \\ -x & 4 \end{vmatrix} = 34$ $x = -\frac{7}{2}$ or $-3.5$

15. $\begin{vmatrix} 3 & -1 \\ 3 & 4x \end{vmatrix} = 21$ $x = \frac{3}{2}$ or $1.5$

**Solve the system using matrices. Write the solutions as ordered pairs.**

16. $\begin{aligned} 2x+3y&=7 \\ 4x-4y&=4 \end{aligned}$ $(2,1)$

17. $\begin{aligned} -5x-y&=2 \\ 10x+3y&=1 \end{aligned}$ $\left(-\frac{7}{5}, 5\right)$

**Transform Polygon ABCD with vertices A(-3, 2) B(-5, 4) C(4, 3) and D(3, -1) the following ways.**

18. Translate 2 units right and 5 units down. $\begin{bmatrix} -1 & -3 & 6 & 5 \\ -3 & -1 & -2 & -6 \end{bmatrix}$

19. Reflect over the y-axis $\begin{bmatrix} 3 & 5 & -4 & -3 \\ 2 & 4 & 3 & -1 \end{bmatrix}$

20. Rotate 90° Counterclockwise $\begin{bmatrix} -2 & -4 & -3 & 1 \\ -3 & -5 & 4 & 3 \end{bmatrix}$

**CALCULATOR SECTION**

**Find the inverse if it exists.**

21. $\begin{bmatrix} -7 & -1 & 2 \\ 3 & 6 & 4 \\ 0 & 11 & -2 \end{bmatrix} = \begin{bmatrix} \frac{-19}{113} & \frac{5}{113} & \frac{-4}{113} \\ \frac{3}{226} & \frac{7}{226} & \frac{17}{226} \\ \frac{33}{452} & \frac{77}{452} & \frac{-39}{452} \end{bmatrix}$

22. **Write as a matrix equation & solve.**

$\begin{aligned} 3x+4y+2z&=12 \\ -2x-3y-4z&=-12 \\ 5x+5y+6z&=8 \end{aligned}$ $\begin{aligned} x&=-7 \\ y&=8 \\ z&=.5 \end{aligned}$

23. **Write as a matrix equation & solve.**

$\begin{aligned} 2x+z&=6 \\ 3x-2y+4z&=13 \\ -y-3z&=-15 \end{aligned}$ $\begin{aligned} x&=1 \\ y&=3 \\ z&=4 \end{aligned}$

24. **The Arcadium arcade in Lynchburg, Tennessee uses 3 different colored tokens for their game machines. For $20 you can purchase any of the following mixtures of tokens: 14 gold, 20 silver, and 24 bronze; OR, 20 gold, 15 silver, and 19 bronze; OR, 30 gold, 5 silver, and 13 bronze. What is the monetary value of each token?**

Gold: $.50, Silver: $.35, Bronze: $.25

26. **Using matrices multiplication, calculate the following grade for the students using the tables below.**

| Student | Tests | Projects | Homework | Quizzes |
|---|---|---|---|---|
| Alexandra | 92 | 100 | 89 | 80 |
| Megan | 72 | 85 | 80 | 75 |
| Brittney | 88 | 78 | 85 | 92 |

| Type | Weight |
|---|---|
| Tests | 40% (.4) |
| Projects | 15% (.15) |
| Homework | 25% (.25) |
| Quizzes | 20% (.2) |

Alexandra: 90%
Megan: 77%
Brittney: 87%

# Unit 5 Review

1. $\det \begin{bmatrix} 2 & 1 & 1 \\ 7 & 4 & -3 \\ -1 & 5 & 1 \end{bmatrix}$ $\begin{matrix} 2 & 1 \\ 7 & 4 \\ -1 & 5 \end{matrix}$

$8+3+35-(-4-30+7)$

$46-(-27)=\boxed{73}$

2. $\det \begin{bmatrix} 10 & 4 & 6 \\ 2 & -3 & 1 \\ -3 & 2 & 0 \end{bmatrix}$ $\begin{matrix} 10 & 4 \\ 2 & -3 \\ -3 & 2 \end{matrix}$

$0-12+24-(54+20+0)$

$12-(74)=\boxed{-62}$

3. $\begin{bmatrix} -1 & 4 & -2 \\ 1 & -1 & -4 \end{bmatrix} \begin{bmatrix} -6 & 2 & -5 \\ 1 & -4 & 2 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 6+4-6 & -2-16+0 & 5+8+2 \\ -6-1-12 & 2+4+0 & -5-2+4 \end{bmatrix}$

$\begin{bmatrix} 4 & -18 & 15 \\ -19 & 6 & -3 \end{bmatrix}$

4. $\begin{bmatrix} 21 & 12 \\ 7 & 4 \end{bmatrix}$

$\det A = 84-84=0$

no inverse exists

5. $A=\frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ -1 & -7 & 1 \\ 4 & 11 & 1 \end{vmatrix} \begin{matrix} 3 & -2 \\ -1 & -7 \\ 4 & 11 \end{matrix} = \frac{1}{2}(-21-8-11-(-28+33+2))$

$A=\frac{1}{2}(-40-(7))=\frac{1}{2}(-47)=\boxed{23.5}$

6. $\begin{bmatrix} 3 & 2 & 1 \\ -5 & 6 & -3 \end{bmatrix}$ $2\times3$

7. $\begin{bmatrix} 8 \\ 9 \\ -2 \\ 3 \end{bmatrix}$ $4\times1$

8. $\begin{bmatrix} 2 & 43 \end{bmatrix}$ $1\times2$

9. $-5\begin{bmatrix} 6 & 2 & 0 \\ 4 & -11 & 6 \end{bmatrix} = \begin{bmatrix} -30 & -10 & 0 \\ -20 & 55 & -30 \end{bmatrix}$

10. $4\begin{bmatrix} -8 & 10 \\ 0 & -4 \end{bmatrix} - 5\begin{bmatrix} -1 & 4 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} -32 & 40 \\ 0 & -16 \end{bmatrix} + \begin{bmatrix} 5 & -20 \\ -20 & -35 \end{bmatrix}$

$= \begin{bmatrix} -27 & 20 \\ -20 & -51 \end{bmatrix}$

11. $\begin{bmatrix} -6 & 9 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 5 & -2 & 4 \\ 1 & 0 & -5 \end{bmatrix} = \begin{bmatrix} -30+9 & 12+0 & -24-45 \\ 10-1 & -4+0 & 8+5 \end{bmatrix}$

$= \begin{bmatrix} -21 & 12 & -69 \\ 9 & -4 & 13 \end{bmatrix}$

12. $\begin{bmatrix} 7 & 0 & -8 \\ 10 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ 4 & 0 \end{bmatrix}$ = undefined

13. $8\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} + 3\begin{bmatrix} 4 \\ -6 \\ 7 \end{bmatrix} - \begin{bmatrix} -1 \\ 8 \\ -6 \end{bmatrix} = \begin{bmatrix} 16 \\ -8 \\ 32 \end{bmatrix} + \begin{bmatrix} 12 \\ -18 \\ 21 \end{bmatrix} + \begin{bmatrix} 1 \\ -8 \\ 6 \end{bmatrix} = \begin{bmatrix} 29 \\ -34 \\ 59 \end{bmatrix}$

14. $\begin{vmatrix} 5 & -4 \\ -x & 4 \end{vmatrix} = 34$ $\quad 20 - 4x = 34$

$-4x = 14$

$x = -\frac{7}{2}$ or $-3.5$

15. $\begin{vmatrix} 3 & -1 \\ 3 & 4x \end{vmatrix} = 21$ $\quad 12x + 3 = 21$

$12x = 18$

$x = \frac{3}{2}$ or $1.5$

16. $\begin{bmatrix} 2 & 3 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$

$A^{-1} = \frac{1}{-20}\begin{bmatrix} -4 & -3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{3}{20} \\ \frac{1}{5} & -\frac{1}{10} \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{3}{20} \\ \frac{1}{5} & -\frac{1}{10} \end{bmatrix}\begin{bmatrix} 7 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\quad \boxed{(2, 1)}$

17. $\begin{bmatrix} -5 & -1 \\ 10 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$A^{-1} = \frac{1}{-5}\begin{bmatrix} 3 & 1 \\ -10 & -5 \end{bmatrix} = \begin{bmatrix} -\frac{3}{5} & -\frac{1}{5} \\ 2 & 1 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{3}{5} & -\frac{1}{5} \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{7}{5} \\ 5 \end{bmatrix}$ $\quad \boxed{(-\frac{7}{5}, 5)}$

18. $\begin{bmatrix} 2 & 2 & 2 & 2 \\ -5 & -5 & -5 & -5 \end{bmatrix} + \begin{bmatrix} -3 & -5 & 4 & 3 \\ 2 & 4 & 3 & -1 \end{bmatrix} = \begin{bmatrix} -1 & -3 & 6 & 5 \\ -3 & -1 & -2 & -6 \end{bmatrix}$

19. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -3 & -5 & 4 & 3 \\ 2 & 4 & 3 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 5 & -4 & -3 \\ 2 & 4 & 3 & -1 \end{bmatrix}$

20. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -3 & -5 & 4 & 3 \\ 2 & 4 & 3 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -4 & -3 & 1 \\ -3 & -5 & 4 & 3 \end{bmatrix}$

21. $A = \begin{bmatrix} -7 & -1 & 2 \\ 3 & 6 & 4 \\ 0 & 11 & -2 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} -\frac{14}{113} & \frac{5}{113} & -\frac{4}{113} \\ \frac{3}{226} & \frac{7}{226} & \frac{17}{226} \\ \frac{33}{452} & \frac{77}{452} & -\frac{39}{452} \end{bmatrix}$

22. $\begin{bmatrix} 3 & 4 & 2 \\ -2 & -3 & -4 \\ 5 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -12 \\ 8 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}B = \begin{bmatrix} -7 \\ 8 \\ .5 \end{bmatrix}$ or $\begin{bmatrix} -7 \\ 8 \\ 1/2 \end{bmatrix}$ $(-7, 8, .5)$

23. $\begin{bmatrix} 2 & 0 & 1 \\ 3 & -2 & 4 \\ 0 & -1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 13 \\ -15 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}B = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$ $(1, 3, 4)$

24. G = $ of gold, S = $ of silver, B = $ of bronze

$14G + 20S + 24B = 20$

$20G + 15S + 19B = 20$

$30G + 5S + 13B = 20$

$$\begin{bmatrix} 14 & 20 & 24 \\ 20 & 15 & 19 \\ 30 & 5 & 13 \end{bmatrix} \begin{bmatrix} G \\ S \\ B \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \\ 20 \end{bmatrix}$$

$$\begin{bmatrix} G \\ S \\ B \end{bmatrix} = A^{-1}B = \begin{bmatrix} .5 \\ .35 \\ .25 \end{bmatrix}$$

Gold tokens are $.50 each, silver are $.35 each, and bronze are $.25 each.

25.

$$\begin{array}{c} \\ A \\ M \\ B \end{array} \begin{array}{c} \begin{array}{cccc} T & P & H & Q \end{array} \\ \begin{bmatrix} 92 & 100 & 89 & 80 \\ 72 & 85 & 80 & 75 \\ 88 & 78 & 85 & 92 \end{bmatrix} \end{array} \cdot \begin{array}{c} \text{Weight} \\ \begin{array}{c} T \\ P \\ H \\ Q \end{array} \begin{bmatrix} .4 \\ .15 \\ .25 \\ .2 \end{bmatrix} \end{array} = \begin{array}{c} \text{Grade} \\ \begin{array}{c} A \\ M \\ B \end{array} \begin{bmatrix} 90.05 \\ 76.55 \\ 86.55 \end{bmatrix} \end{array}$$

Alexandra has a 90%, Megan has a 77%, and Brittney has an 87%