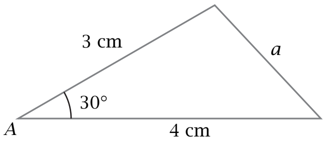
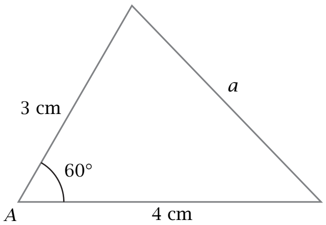
Name: ______________________ Group Members: ______________________

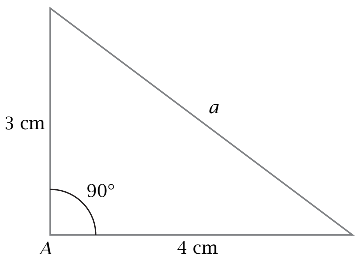
Exploration 6-1a: Introduction to Oblique Triangles

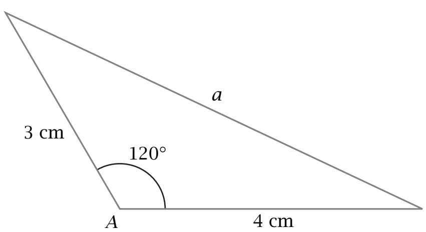
Date: ____________

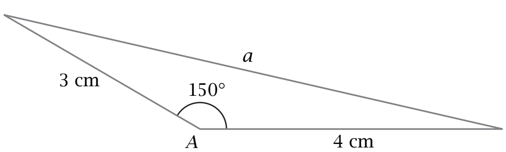
Objective: Given the lengths of two sides of a triangle and the measure of the included angle, measure the third side.











1. The figures show five triangles. Each has sides of 3 cm and 4 cm. They differ in the size of the angle included between the two sides. Are the measurements of the lengths of the sides correct as marked on each figure? ____________

2. Are the degree measurements correct as shown in each figure? ____________

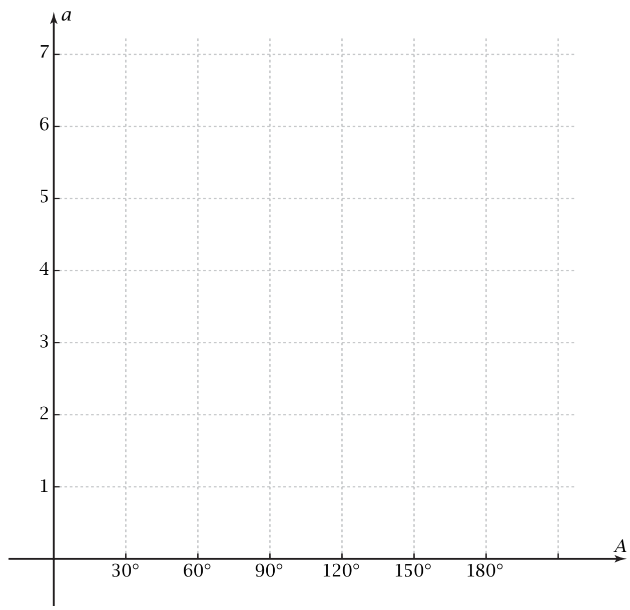
3. Measure the third side, a, of each triangle. Record the results here, correct to one decimal place.

***A* (degrees)**	***a* (cm)**
30°	
60°	
90°	
120°	
150°	

4. What would a equal if $\angle A$ were:

   180°: a = ____________ 0°: a = ____________

5. Plot the measured values of a as a function of the angle measure, A, from 0° to 180°. Connect the points with a smooth curve.



6. By the Pythagorean theorem, $a^2 = 3^2 + 4^2$ when A is 90°. If A is *less* than 90°, you have to *subtract* something to get the value of a^2. Consult your textbook to find out *what* expression must be subtracted.

7. The answer to Problem 6 is part of the **law of cosines.** Use the law of cosines to calculate a for each value of angle A in the table of Problem 3.

8. What did you learn as a result of doing this Exploration that you did not know before?

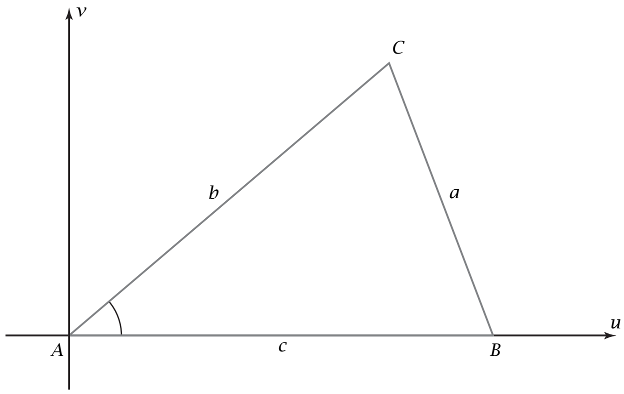
Name: ______________________ Group Members: ______________________

Exploration 6-2a: Derivation of the Law of Cosines

Date: ____________

Objective: Derive the law of cosines for predicting the third side of a triangle from two sides and the included angle.

The figure shows triangle ABC. Angle A has been placed in standard position in a uv-coordinate system.



1. The sides that include angle A have lengths b and c. Write the coordinates of points B and C using b, c, and functions of angle A.

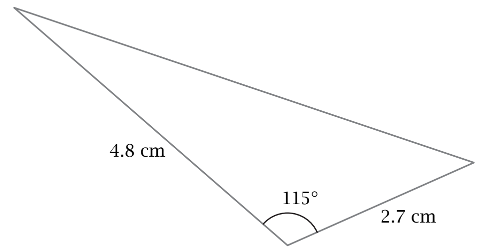
   B: $(u, v) =$ (____________, ____________)

   C: $(u, v) =$ (____________, ____________)

2. Use the **distance formula** to write the square of the length of the third side, a^2, in terms of b, c, and functions of angle A.

3. Simplify the answer to Problem 2 by expanding the square. Use the **Pythagorean property** for cosine and sine to simplify the terms containing $\cos^2 A$ and $\sin^2 A$.

4. The answer to Problem 3 is called the **law of cosines.** Show that you understand what the law of cosines says by using it to calculate the third side of this triangle.



5. Measure the given sides and the angle of the triangle in Problem 4. Do you agree with the given measurements? Measure the third side. Does it agree with your calculated value?

6. Describe how the unknown side in the law of cosines is related to the given angle and how the given angle is related to the two given sides, using terms you studied in geometry.

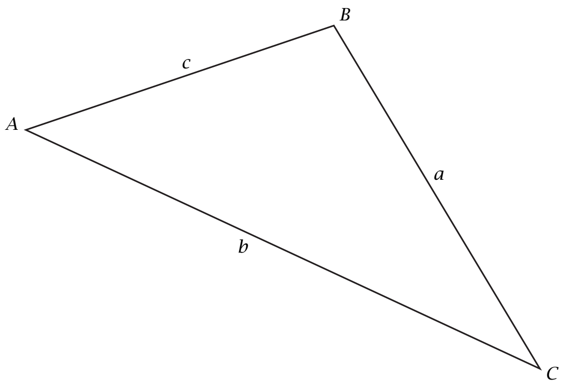
7. What have you learned as a result of doing this Exploration that you did not know before?

Name: ______________________ Group Members: ______________________

Exploration 6-2b: Angles by Law of Cosines

Date: __________

Objective: Use the law of cosines to calculate the measure of an angle of a triangle if three sides are known.



1. For $\triangle ABC$, write the law of cosines in the form involving cos *B*.

   ____

2. Measure the sides of $\triangle ABC$ to the nearest 0.1 cm.

   $a \approx$ ________ $b \approx$ ________ $c \approx$ ________

3. Substitute the values of *a*, *b*, and *c* from Problem 2 into the law of cosines from Problem 1. Solve the resulting equation for *B*. Store the answer without rounding.

4. Measure angle *B* with a protractor. ____________

5. How close did your calculated angle value come to the measured value?

6. Without substituting values for *a*, *b*, and *c*, solve the equation from Problem 1 for cos *B* in terms of *a*, *b*, and *c*. Simplify as much as possible.

7. By observing the pattern in your answer to Problem 6, write an equation for cos *A* in terms of *a*, *b*, and *c*.

8. Use your answer to Problem 7 and the measured values of *a*, *b*, and *c* to calculate the measure of angle *A*. Store the answer without rounding.

9. Use the pattern in Problem 6 to calculate the measure of angle *C* by the law of cosines. Store the answer without rounding.

10. Use the stored values of *A*, *B*, and *C* to show that the sum of the angles in the triangle is 180°.

11. Measure angles *A* and *C*.

    $A \approx$ ____________ $C \approx$ ____________

12. How close do your calculated angle values come to the measured values?

13. What did you learn as a result of doing this Exploration that you did not know before?

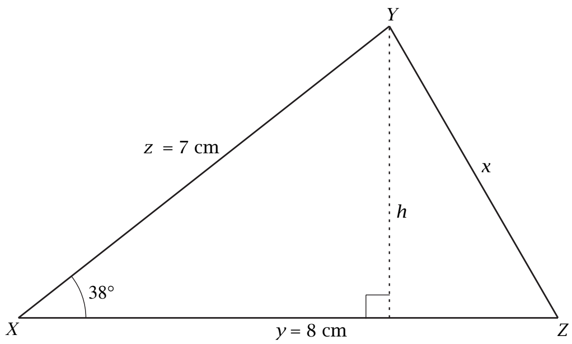
Name: ____________________ Group Members: ____________________

Exploration 6-3a: Area of a Triangle and Hero's Formula

Date: ____________

Objective: Derive a *quick* method to calculate the area of a triangle from two sides and the included angle.

For Problems 1–3, $\triangle XYZ$ has sides $y = 8$ cm, $z = 7$ cm, and included angle $X = 38°$.



1. Do you agree with the given measurements?

   y ________ z ________ $\angle X$ ________

2. Use the given measurements to calculate altitude h. Measure h. Does it agree with the calculation?

3. You recall from geometry that the area of a triangle is $\frac{1}{2}$(base)(altitude). Find the area of $\triangle XYZ$.

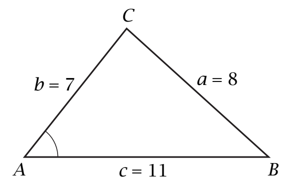
4. By substituting $z \sin X$ in Problem 3 you get

   Area $= \dfrac{1}{2}yz \sin X$, or, in general,

   Area $= \dfrac{1}{2}$(side)(side)(sine of included angle).

   Sketch a triangle with sides 43 m and 51 m, and included angle 143°. Use this area formula to find the area of this triangle.

For Problems 5–8, $\triangle ABC$ has sides $a = 8$, $b = 7$, and $c = 11$.



5. Find the measure of angle A using the law of cosines. Store the answer without rounding.

6. Use the unrounded value of A and the area formula of Problem 3 to find the area of $\triangle ABC$.

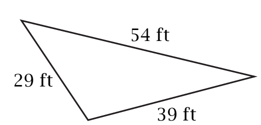
7. Calculate the **semiperimeter** (half the perimeter) of the triangle, $s = \frac{1}{2}(a + b + c)$.

8. Evaluate the quantity $\sqrt{s(s - a)(s - b)(s - c)}$. What interesting thing do you notice about the answer?

9. Use **Hero's formula,** namely,

   Area $= \sqrt{s(s - a)(s - b)(s - c)}$

   to find the area of this triangle.



10. What did you learn as a result of doing this Exploration that you did not know before?

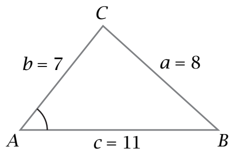
Name: ______________________ Group Members: ______________________________

Exploration 6-3b: Derivation of Hero's Formula

Date: __________

Objective: Derive Hero's formula for calculating the area of a triangle given the measures of the three sides.

The figure shows $\triangle ABC$ with sides $a = 8$, $b = 7$, and $c = 11$.



1. Calculate the area of $\triangle ABC$ again using **Hero's formula:**

   $$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

   where s is the **semiperimeter** of the triangle (half the perimeter).

2. The area formula from Section 6-3 is

   $$\text{Area} = \frac{1}{2}bc\sin A$$

   Explain why this can be written as

   $$\text{Area} = \sqrt{\frac{1}{4}b^2c^2(1-\cos^2 A)}$$

3. Explain why the result of Problem 2 can be written as

   $$\text{Area} = \sqrt{\frac{1}{2}bc(1+\cos A)\cdot\frac{1}{2}bc(1-\cos A)}$$

4. Use the law of cosines to write $\cos A$ in terms of a, b, and c *without* substituting the values.

5. Show that $\frac{1}{2}bc(1+\cos A)$ can be written as

   $$\frac{(b+c)^2-a^2}{4} = \frac{b+c+a}{2}\cdot\frac{b+c-a}{2}$$

(Over)

6. By substitutions such as you did in Problem 5, it is possible to transform the other factor under the radical in Problem 3 this way. (It is not necessary for you to *do* the transformations.)

$$\frac{1}{2}bc(1 - \cos A) = \frac{a - b + c}{2} \cdot \frac{a + b - c}{2}$$

Using this information and the result of Problem 5, the area can be written as

$$\text{Area} = \sqrt{\frac{b + c + a}{2} \cdot \frac{b + c - a}{2} \cdot \frac{a - b + c}{2} \cdot \frac{a + b - c}{2}}$$

The first fraction under the radical is the semiperimeter, $s = \frac{1}{2}(a + b + c)$. Show that the other three fractions can be written as $(s - a)$, $(s - b)$, and $(s - c)$, respectively. From the results, derive Hero's formula,

$$\text{Area} = \sqrt{s(s - a)(s - b)(s - c)}$$

7. Use Hero's formula to find the area of $\triangle XYZ$ if $x = 50$ cm, $y = 60$ cm, and $z = 80$ cm.

8. Show that Hero's formula indicates that there is no possible $\triangle BAD$ if $b = 10$ ft, $a = 12$ ft, and $d = 26$ ft.

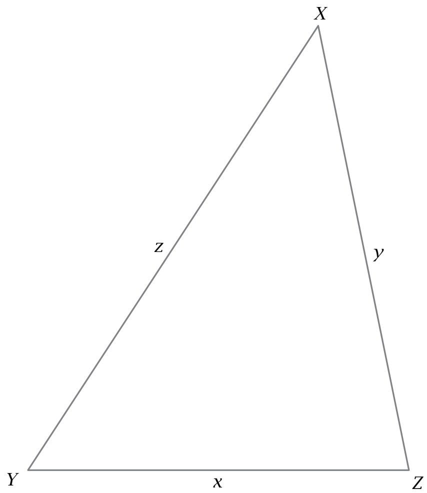
9. What did you learn as a result of doing this Exploration that you did not know before?

Name: ______________________ Group Members: ______________________

Exploration 6-4a: The Law of Sines

Date: ____________

Objective: Use the ratio of a side length to the sine of the opposite angle to find other parts of a triangle.



1. In $\triangle XYZ$, are the following measurements correct?

   $y = 6.0$ cm ________ $z = 7.0$ cm ________

   $Y = 57°$ ________ $Z = 78°$ ________

2. Assuming that the measurements in Problem 1 are correct, calculate these ratios:

   $\frac{y}{\sin Y} =$ ______________

   $\frac{z}{\sin Z} =$ ______________

3. The **law of sines** states that within a triangle, the ratio of the length of a side to the sine of the opposite angle is constant. Do the calculations in Problem 2 seem to confirm this property? ________

4. Measure angle X. ______________

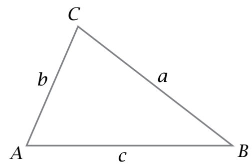
5. Assuming that the law of sines is correct,

   $$\frac{x}{\sin X} = \frac{y}{\sin Y}$$

   Use this information and the measured value of X to calculate the length x.

6. Measure side x. Does your measurement agree with the calculated value in Problem 5? ________

The law of sines can be derived **algebraically.**



7. For $\triangle ABC$, use the area formula to write the area *three* ways: one involving angle A, one involving angle B, and one involving angle C.

   ____

   ____

   ____

8. The area of a triangle is *independent* of the way you *measure* that area, so all three area expressions in Problem 7 are equal to each other. Write a *three*-part equation expressing this fact.

9. Divide all three "sides" of the equation in Problem 8 by whatever is necessary to leave only the sines of the angles in the numerators. Simplify.

10. The equation you should have gotten in Problem 9 is the **law of sines.** Explain why it is equivalent to the law of sines as written in Problem 5.

11. What did you learn as a result of doing this Exploration that you did not know before?

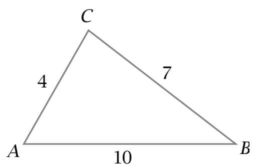
Name: ______________________ Group Members: ______________________

Exploration 6-4b: The Law of Sines for Angles

Date: ____________

Objective: Discover the hazards of using the law of sines to find an angle of a triangle.

The figure shows $\triangle ABC$ (not to scale) with $a = 7$, $b = 4$, and $c = 10$.



1. Use the law of cosines to find the measure of angle A. Store the answer in your calculator without round-off.

2. Use the answer to Problem 1 and the law of sines to calculate the measure of angle C.

3. Calculate the measure of angle C again, directly from the three side lengths, using the law of cosines.

4. Do your answers to Problems 2 and 3 agree? If not, describe how you can use the *general* solution for inverse sine (arcsin instead of $\sin^{-1}$) to get the correct answer by the law of sines.

5. Triangle XYZ has $x = 11$ in., $y = 8$ in., and angle $Y = 40°$. Use the law of sines to find *two* possible values for the measure of angle X. Sketch both triangles.

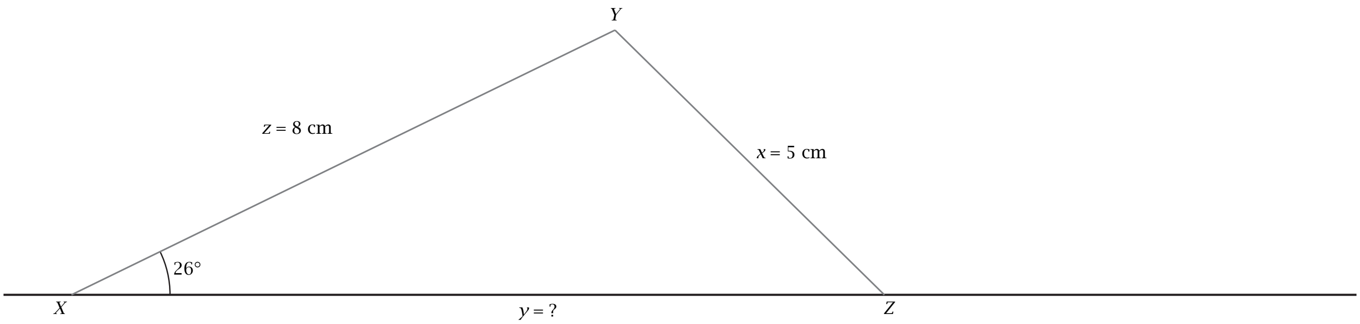
6. What did you learn as a result of doing this Exploration that you did not know before?

Name: ____________________ Group Members: ____________________

Exploration 6-5a: The Ambiguous Case, SSA

Date: __________

Objective: Investigate what can happen if SSA (side, side, angle) is given in a triangle.



1. In $\triangle XYZ$, side $x = 5$ cm, side $z = 8$ cm, and angle $X = 26°$. Do you agree with these measurements?

   ____

2. Draw another possibility for $\triangle XYZ$ with the same values of x, z, and X but a different value of y. For both triangles, measure side y.

   $y =$ ____________ or $y =$ ____________

3. What does the word **ambiguous** mean? Why is case SSA called *ambiguous?*

4. If side z and angle X remain fixed and side x is increased to 9 cm, will there still be two possible triangles? Draw the result on the given figure.

5. If side z and angle X remain fixed and side x is decreased to 3 cm, how many possible triangles will there be? Show your conclusion on the given figure.

6. When side x is perpendicular to side y, there is exactly one triangle. Calculate the value of x in this case. Confirm that it is correct by taking its measurement on the figure.

7. By clever application of the law of cosines and the quadratic formula, it is possible to *calculate* both values of y in Problem 2. Do this calculation.

8. Show how the technique of Problem 7 justifies the answer to Problem 4.

9. Show how the technique of Problem 7 justifies the answer to Problem 5.

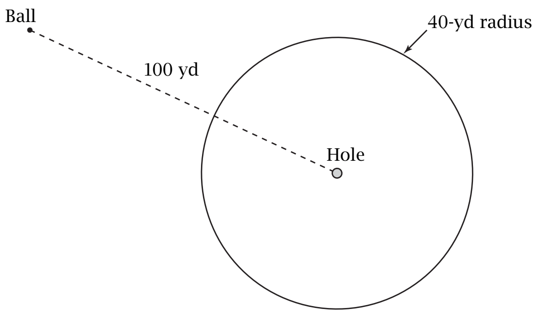
10. What did you learn as a result of doing this Exploration that you did not know before?

Name: ______________________ Group Members: ______________________

Exploration 6-5b: Golf Ball Problem

Date: __________

Objective: Analyze a real-world problem involving the ambiguous case.

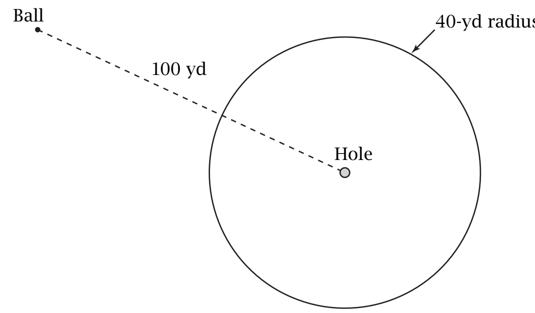


Golf Ball Problem: Dolph Ball hits his tee shot. The ball winds up exactly 100 yd from the hole. On his second shot, the ball winds up exactly 40 yd from the hole, somewhere on the circle shown in the figure.

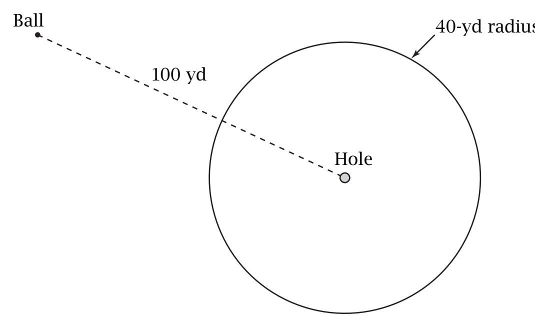
1. If Dolph's second shot went on a line 15° to the right of the line to the hole, plot on the figure the path the ball took. Show one point where the path crosses the circle. Then draw a triangle with the distance the ball traveled to get to this point as one of its sides and the 40 yd and 100 yd as the other two sides.

2. Use the law of cosines to calculate the *two* possible distances the shot in Problem 1 could have gone. Show both distances on the figure.

3. If Dolph's second shot had gone on a line 30° to the right of the line to the hole, plot the path of the ball on this copy of the figure. Show by calculation that the ball could not have come within 40 yd of the hole.



4. Show on this copy of the figure the path of the ball at the maximum angle Dolph's second shot could have made and still have come to rest 40 yd from the hole. Calculate the measure of this angle. Does the angle you drew have this measure? __________



5. What did you learn as a result of working this Exploration that you did not know before?

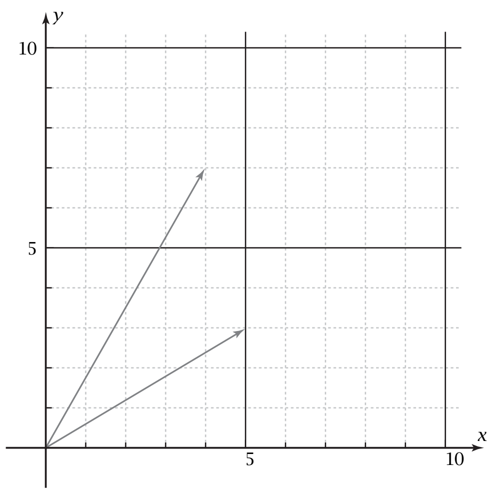
Name: ______________________ Group Members: ______________________________________

Exploration 6-6a: Introduction to Vectors

Date: ____________

Objective: Use the properties of triangles to add vectors.

1. The figure shows two vectors starting from the origin. One ends at the point (4, 7) and the other ends at (5, 3). Translate one of the two vectors so that the vectors are in position to be added. Then draw the resultant vector—the sum of the two vectors.



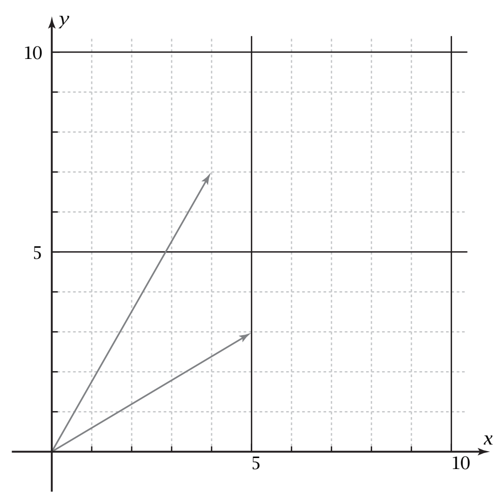
2. Calculate the length of the resultant vector in Problem 1 and the angle it makes with the *x*-axis.

3. The two given vectors and the resultant vector form a triangle. Calculate the measure of the largest angle in this triangle.

4. Calculate the measure of the angle between the two vectors when they are placed tail-to-tail, as they were given in Problem 1.

5. In Problem 1, you translated one of the vectors. Show on the figure that you would have gotten the *same* resultant vector if you had translated the *other* vector. Use a different color than you used in Problem 1.

6. The vectors in Problem 1 have **components** in the *x*-direction and in the *y*-direction. These components are vectors that can be added together to equal the given vector. On this copy of the figure, show how the components of the longer vector can be added to give that vector.



7. Give an *easy* way to get the components of the sum of the two given vectors in Problem 1.

8. What did you learn as a result of doing this Exploration that you did not know before?

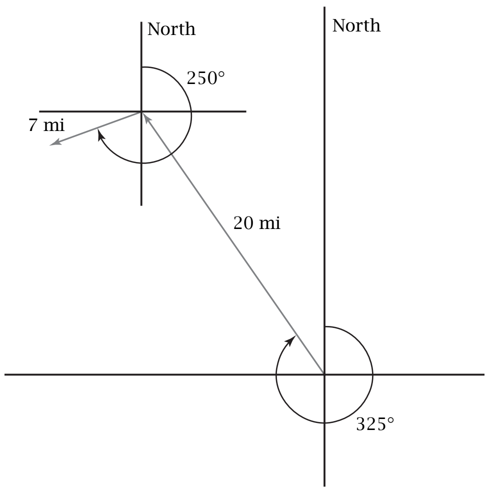
Name: ______________________ Group Members: ______________________

Exploration 6-6b: Navigation Vectors

Date: ____________

Objective: Work navigation problems using components of vectors.

A ship sails for 20 miles along a bearing of $\beta_1 = 325°$ and then turns and sails on a bearing of $\beta_2 = 250°$ for 7 more miles, as shown in the figure.



1. Bearings are measured clockwise from North. Show bearings of 0°, 90°, 180°, and 270° on the figure. What bearing is equivalent to 360°?

2. Assume that the unit vectors $\vec{i}$ and $\vec{j}$ point along bearings of 0° and 90°, the same as they would for angles in standard position. Use this fact to write components for the two given vectors.

3. Find the $\vec{i}$ and $\vec{j}$ components for the resultant displacement vector.

4. Find bearing β for the resultant vector.

5. Find the magnitude of the resultant vector. Then write the vector as a magnitude and bearing.

A ship sails with a velocity of 20 knots (nautical miles per hour) on a bearing of 325°. The water has a current of velocity 7 knots on a bearing of 250°.

6. Draw a diagram showing the two vectors coming from the origin.

7. The ship's resultant velocity is the vector sum of the two velocity vectors. Find this resultant velocity vector.

8. What did you learn as a result of doing this Exploration that you did not know before?

Name: ______________________ Group Members: ______________________

Exploration 6-7a: The Ship's Path Problem

Date: ____________

Objective: Work a real-world triangle problem given only the descriptive words.

Suppose that you are on a rescue mission in the Gulf of Suez. Your ship is steaming east when its sonar detects the disabled submarine 6 (nautical) miles away, at an angle of 28° to the south of the easterly path.

1. Construct a diagram showing the rescue ship, its easterly path, and the submarine southeast of the ship. Use 1 cm for 1 nautical mile.

2. The submarine can transmit distress signals that can be detected at a 4-mile radius. Calculate how far your rescue ship would have had to go on its easterly path before the distress signals would have reached it had it not detected the submarine.

3. Plot the distance you calculated in Problem 2 on the diagram from Problem 1. Is the point really 4 (scale) miles from the submarine? ____________

4. Assume that your rescue ship missed the distress signal the first time it could have been detected and continues on its easterly path. Calculate the farthest your ship can be from the initial point in Problem 2 while still able to hear the distress signal.

5. Calculate the angle at the submarine between the rescue ship's initial position in Problem 1 and its position in Problem 4.

6. Find the area of the triangular region of ocean with vertices at the submarine, the ship's initial Problem 1 position, and its Problem 4 position.

7. The submarine also carries short-range radio transmitters that will transmit radio signals only 2 miles. Was your ship ever within range of these radio signals? Show how you reach your conclusion.

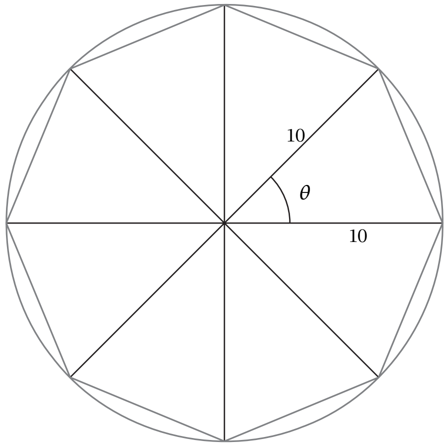
8. What did you learn as a result of doing this Exploration that you did not know before?

Name: ______________________ Group Members: ______________________

Exploration 6-7b: Area of a Regular Polygon

Date: ____________

Objective: Apply your knowledge of triangle trigonometry to a new situation.



1. The figure shows a regular octagon inscribed in a circle of radius 10 units. Diagonals connecting the opposite vertices divide the octagon into eight triangles meeting at the center. Calculate the measure of central angle θ. Use the result and the appropriate triangle property to calculate the area of one triangle. Then find the area of the octagon.

2. Suppose a regular *n*-gon (an *n*-sided polygon) is inscribed in the circle shown. Write an equation expressing the area of an *n*-gon as a function of *n*.

3. Enter the equation of Problem 2 in the y= menu of your grapher. Make a table of values of area for *n*-gons, starting with a triangle.

4. There are two of these regular polygons that have *integer* areas. Which ones? What are the areas?

5. Write a short program to calculate and display the areas of each *n*-gon, starting at $n = 3$ and going to at least $n = 600$. Set the mode so that your grapher displays a fixed number of digits, at least nine. Then run the program.

6. As the program is running, what do you notice is happening to more and more digits in the areas?

7. The areas are approaching a **limit** as *n* gets larger and larger. What do you think it means for a quantity to approach a limit? What number does that limit equal? Explain geometrically why that number is reasonable.

8. What did you learn as a result of doing this Exploration that you did not know before.

25. $\sqrt{3}\cos x - \sin x = 2\cos\left(x + \dfrac{\pi}{6}\right)$

26. $4\cos x - 4\sin x = 4\cos\left(x + \dfrac{\pi}{4}\right)$

27. Answers will vary.

Chapter 6: Triangle Trigonometry

Exploration 6-1a

1. Measurements are correct.

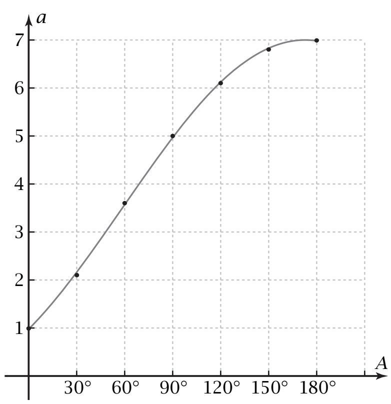
2. Measurements are correct.

3. Answers will vary slightly but should be approximately

$\angle A$	a (cm)
30°	2.1
60°	3.6
90°	5.0
120°	6.1
150°	6.8

4. 180°: $a = 4 + 3 = 7$ cm
   0°: $a = 4 - 3 = 1$ cm

5.



6. Answers may vary. The actual answer is $24\cos\theta$.

7.

$\angle A$	a (cm)
30°	2.0531...
60°	3.6055...
90°	5.0000...
120°	6.0828...
150°	6.7664...

8. Answers will vary.

Exploration 6-2a

1. $(c, 0)$; $(b\cos A, b\sin A)$

2. $a^2 = (b\cos A - c)^2 + (b\sin A - 0)^2$
   $= b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A$

3. $a^2 = b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A$
   $= b^2 + c^2 - 2bc\cos A$

4. $\sqrt{4.8^2 + 2.3^2 - 2\cdot 4.8\cdot 2.7\cos 115^\circ} = 6.4252\ldots$ cm

5. Measurements are correct.

6. The unknown side is opposite the given angle. The two given sides include the given angle.

7. Answers will vary.

Exploration 6-2b

1. $b^2 = a^2 + c^2 - 2ac\cos B$

2. $a = 5.3$ cm; $b = 7.5$ cm; $c = 4.3$ cm

3. $B = \cos^{-1}\dfrac{5.3^2 + 4.3^2 - 7.5^2}{2(5.3)(4.3)} = 102.2486\ldots^\circ \approx 102^\circ$

4. Answers should be close to 102°.

5. Answers may vary.

6. $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

7. $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

8. $A = \cos^{-1}\dfrac{7.5^2 + 4.3^2 - 5.3^2}{2(7.5)(4.3)} = 43.6760\ldots^\circ \approx 44^\circ$

9. $C = \cos^{-1}\dfrac{5.3^2 + 7.5^2 - 4.3^2}{2(5.3)(7.5)} = 37.0753\ldots^\circ \approx 34^\circ$

10. $102.2486\ldots^\circ + 43.6760\ldots^\circ + 34.0753\ldots^\circ = 180^\circ$

11. Answers should be close to $A = 44^\circ$ and $C = 34^\circ$.

12. Answers may vary.

13. Answers will vary.

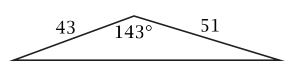
Exploration 6-3a

1. Answers should agree.

2. $h = 7\sin 38^\circ = 4.3096\ldots$
   By measurement, $h \approx 4.3$ cm, which agrees.

3. Area $= \frac{1}{2}(8)(4.3096\ldots) = 17.2385\ldots \approx 17.24$ cm^2

4. Area $= \frac{1}{2}(43)(51)\sin 143^\circ = 659.8901\ldots \approx 659.9$ m^2



5. $\cos A = \dfrac{7^2 + 11^2 - 8^2}{2\cdot 7\cdot 11} = \dfrac{106}{154} = 0.6883\ldots$

   $A = 46.5033\ldots^\circ$

6. Area $= \frac{1}{2}(7)(11)\sin 46.5033\ldots^\circ = 27.9284\ldots$

7. $s = \frac{1}{2}(7 + 11 + 8) = 13$

8. $\sqrt{13(13-7)(13-11)(13-8)} = 27.9284\ldots$

   The answer is the same as the area calculated in Problem 6.

9. $s = \frac{1}{2}(29 + 39 + 54) = 61$
   Area $= \sqrt{61(61-29)(61-39)(61-54)} = \sqrt{300608}$
   $= 548.2773\ldots \approx 548.3$ ft^2

10. Answers will vary.

Exploration 6-3b

1. $s = \frac{1}{2}(7 + 11 + 8) = 13$
   Area $= \sqrt{13(13 - 7)(13 - 11)(13 - 8)} = 27.9284\ldots$

2. Area $= \frac{1}{2}bc \sin A = \sqrt{\left(\frac{1}{2}bc \sin A\right)^2}$
   $= \sqrt{\frac{1}{4}b^2c^2 \sin^2 A} = \sqrt{\frac{1}{4}b^2c^2 (1 - \cos^2 A)}$

3. $(1 - \cos^2 A) = (1 + \cos A)(1 - \cos A)$
   Area $= \sqrt{\frac{1}{4}b^2c^2 (1 + \cos A)(1 - \cos A)}$
   $= \sqrt{\frac{1}{2}bc (1 + \cos A) \cdot \frac{1}{2}bc (1 - \cos A)}$

4. $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

5. $\frac{1}{2}bc(1 + \cos A) = \frac{1}{2}bc\left(1 + \dfrac{b^2 + c^2 - a^2}{2bc}\right)$
   $= \dfrac{bc}{2} + \dfrac{b^2 + c^2 - a^2}{4} = \dfrac{2bc}{4} + \dfrac{b^2 + c^2 - a^2}{4}$
   $= \dfrac{b^2 + 2bc + c^2 - a^2}{4} = \dfrac{(b + c)^2 - a^2}{4}$
   $= \dfrac{((b + c) + a)((b + c) - a)}{4}$
   $= \dfrac{b + c + a}{2} \cdot \dfrac{b + c - a}{2}$

6. $\dfrac{b + c - a}{2} = \dfrac{b + c + a - 2a}{2} = \dfrac{a + b + c}{2} - a = (s - a)$
   $\dfrac{a - b + c}{2} = \dfrac{a + b + c - 2b}{2} = \dfrac{a + b + c}{2} - b = (s - b)$
   $\dfrac{a + b - c}{2} = \dfrac{a + b + c - 2c}{2} = \dfrac{a + b + c}{2} - c = (s - c)$
   So $\sqrt{\dfrac{b + c + a}{2} \cdot \dfrac{b + c - a}{2} \cdot \dfrac{a - b + c}{2} \cdot \dfrac{a + b - c}{2}}$
   $= \sqrt{s(s - a)(s - b)(s - c)}$

7. $s = \frac{1}{2}(50 + 60 + 80) = 95$ cm
   Area $= \sqrt{95(95 - 50)(95 - 60)(95 - 80)}$
   $= 1498.1238\ldots$ cm^2

8. $s = \frac{1}{2}(10 + 12 + 26) = 24$ cm
   Area $= \sqrt{24(24 - 10)(24 - 12)(24 - 26)} = \sqrt{-8064}$
   Hero's formula results in a negative number under the square root, so there is no possible area.

9. Answers will vary.

Exploration 6-4a

1. Measurements are correct.

2. $\dfrac{6.0}{\sin 57°} = 7.1541\ldots$; $\dfrac{7.0}{\sin 78°} = 7.1563\ldots$

3. Yes, to within measurement error

4. 45°

5. $x = \dfrac{6.0 \sin 45°}{\sin 57°} \approx 5.1$ cm

6. Yes

7. $\frac{1}{2}ab \sin C$; $\frac{1}{2}bc \sin A$; $\frac{1}{2}ac \sin B$

8. $\frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$

9. $\dfrac{1}{abc/2}\left(\frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C\right)$
   $\dfrac{1}{abc}bc \sin A = \dfrac{1}{abc}ac \sin B = \dfrac{1}{abc}ab \sin C$
   $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

10. The statements are equivalent because if the parts of an equation are the same and nonzero, then the reciprocals of the parts of the equation are equal and nonzero.

11. Answers will vary.

Exploration 6-4b

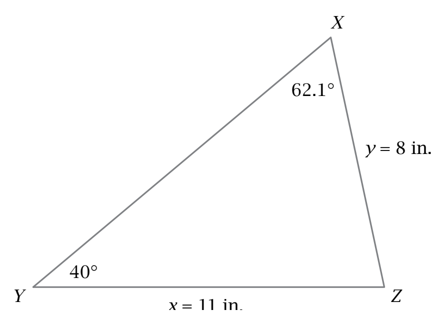
1. $A = \cos^{-1}\dfrac{4^2 + 10^2 - 7^2}{2 \cdot 4 \cdot 10} = 33.12294020\ldots°$

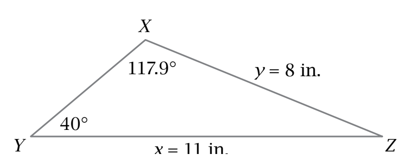
2. $C = \sin^{-1}\dfrac{10 \sin A}{7} = 51.31781254\ldots°$.
   But this is not correct—see Problem 4.

3. $C = \cos^{-1}\dfrac{4^2 + 7^2 - 10^2}{2 \cdot 4 \cdot 7} = 128.68218745\ldots°$

4. No. We should have used
   $180° - 51.31781254\ldots° = 128.68218745\ldots°$, the complement of the answer in Problem 3. ($51.31781254\ldots°$ is what angle C would be if B were to the left of AC, with AB still horizontal.)

5. $X = \sin^{-1}\dfrac{11 \sin 40°}{8} \approx 62.1°$ or $117.9°$





6. Answers will vary.

Exploration 6-5a

1. Measurements are correct.

2. Answers should be close to 10.8 cm and 3.6 cm.

3. "Having two or more possible meanings"; there are two possible answers.

4. Now there is only one answer, 15.5 cm.
   (Student answers should be close to this.)

5. Now there is no answer. Side x is too short.

6. $\dfrac{x}{8} = \sin 26°$, so $x = 8 \sin 26° \approx 3.5$ cm.

7. $5^2 = y^2 + 8^2 - 2 \cdot y \cdot 8 \cos 26°$
$y^2 - 16y \cos 26° + 64 - 25 = 0$
$y^2 + (-16 \cos 26°)y + 39 = 0$
$y = \dfrac{16 \cos 26° \pm \sqrt{(-16 \cos 26°)^2 - 4 \cdot 1 \cdot 39}}{2 \cdot 1}$
≈ 10.8 cm or 3.6 cm

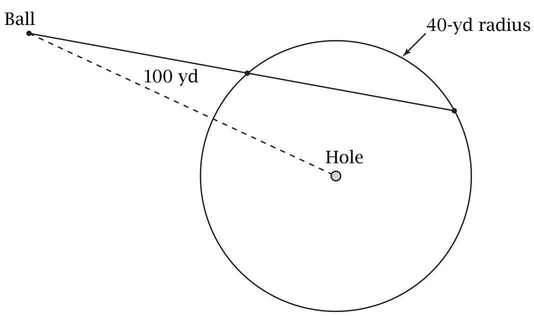
8. $9^2 = y^2 + 8^2 - 2 \cdot y \cdot 8 \cos 26°$
$y^2 - 16y \cos 26° + 64 - 81 = 0$
$y^2 + (-16 \cos 26°)y - 17 = 0$
$y = \dfrac{16 \cos 26° \pm \sqrt{(-16 \cos 26°)^2 - 4 \cdot 1 \cdot (-17)}}{2 \cdot 1}$
≈ 15.5 cm or -1.1 cm (The negative answer represents the triangle that would result if Z were to the left of X.)

9. $3^2 = y^2 + 8^2 - 2 \cdot y \cdot 8 \cos 26°$
$y^2 - 16y \cos 26° + 64 - 9 = 0$
$y^2 + (-16 \cos 26°)y + 55 = 0$
The discriminant $(-16 \cos 26°)^2 - 4 \cdot 1 \cdot 55 \approx -13.2$, so there is no solution.

10. Answers will vary.

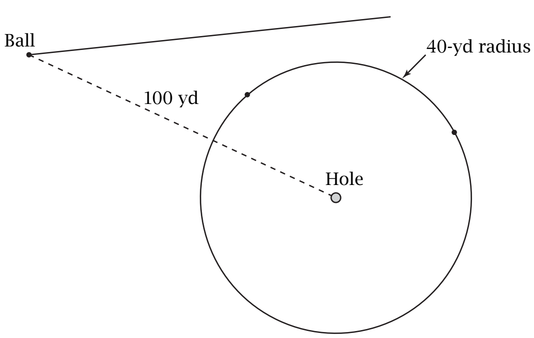
Exploration 6-5b

1.



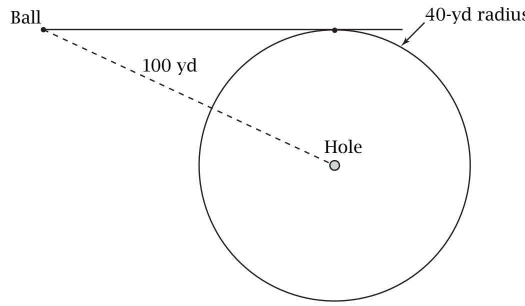
2. Let x be the distance, in feet, the ball traveled.
$40^2 = 100^2 + x^2 - 2 \cdot 100 \cdot x \cdot \cos 15°$
$x^2 - (200 \cos 15°)x + 8400 = 0$
$x = 127.0905\ldots$ or $66.0945\ldots$
Distances are about 127.09 ft and 66.09 ft.

3.



$40^2 = 100^2 + x^2 - 2 \cdot 100 \cdot x \cos 30°$
Discriminant $b^2 - 4ac = -3600$
$\therefore$ no possible values of x.

4.



$\dfrac{40}{100} = \sin\theta$

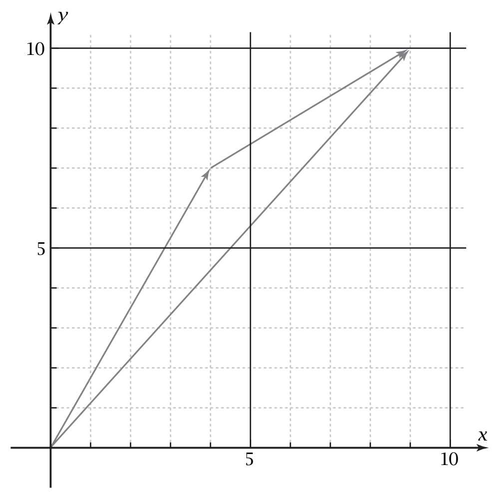
$\sin\theta = \dfrac{2}{5}$

$\theta = 23.578\ldots°$

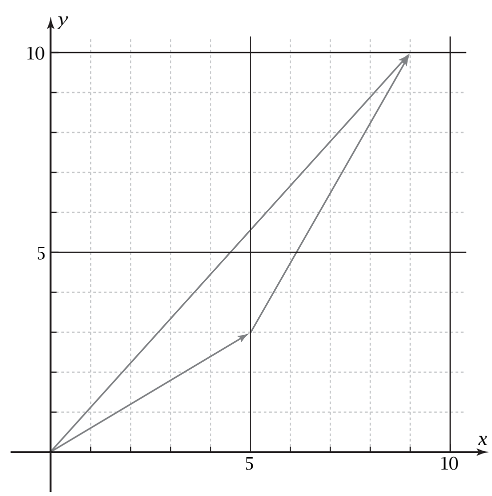
5. Answers will vary.

Exploration 6-6a

1.



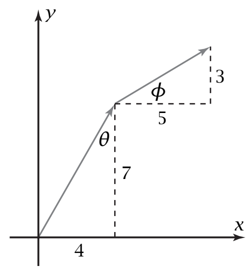
or



2. $|\vec{r}| = \sqrt{9^2 + 10^2} = \sqrt{181} \approx 13.5$

$\theta = \tan^{-1}\dfrac{10}{9} \approx 48.0°$

3.

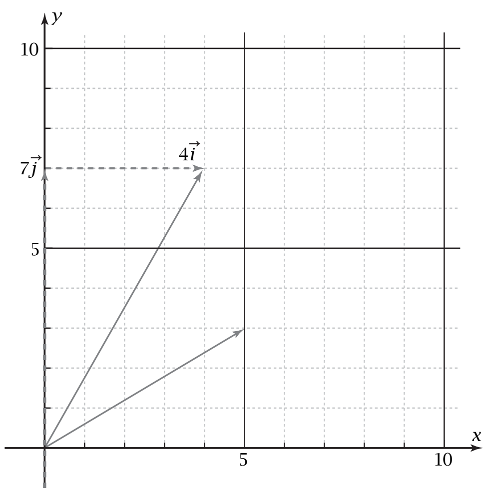


$\theta + 90° + \phi = \tan^{-1}\frac{4}{7} + 90° + \tan^{-1}\frac{3}{5} \approx 150.7°$

4. $\theta - \phi = \tan^{-1}\frac{7}{4} - \tan^{-1}\frac{3}{5} \approx 29.3°$

5. See *both* graphs in Problem 1.

6. $4\vec{i} + 7\vec{j}$

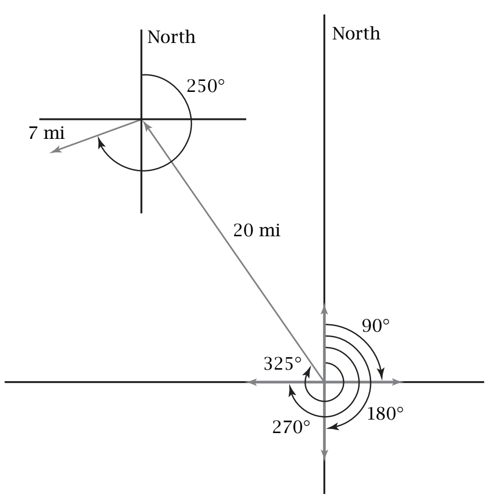


7. The $\vec{i}$-component is the sum of the $\vec{i}$-components, and the $\vec{j}$-component is the sum of the $\vec{j}$-components.

8. Answers will vary.

Exploration 6-6b

1. $0° = 360°$



2. $360° - 325° + 90° = 125°$
$360° - 250° + 90° = 200°$

First vector:
$(20\cos 125°)\vec{i} + (20\sin 125°)\vec{j} \approx -11.5\vec{i} + 16.4\vec{j}$

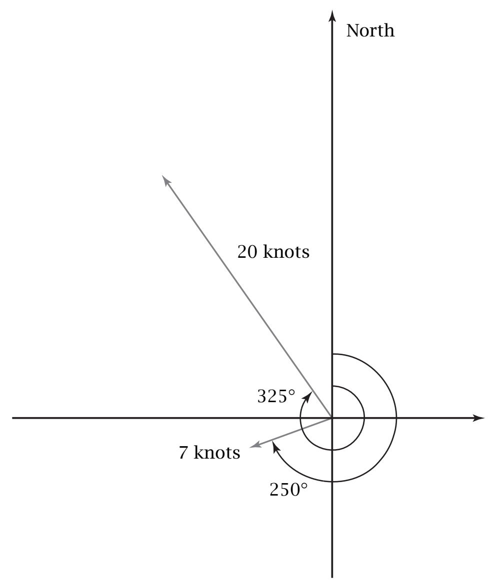
Second vector:
$(7\cos 200°)\vec{i} + (7\sin 200°)\vec{j} \approx -6.6\vec{i} - 2.4\vec{j}$

3. $(20\cos 125° + 7\cos 200°)\vec{i}$
$+ (20\sin 125° + 7\sin 200°)\vec{j} \approx -18.0\vec{i} + 14.0\vec{j}$

4. $\tan^{-1}\dfrac{20\sin 125° + 7\sin 200°}{20\cos 125° + 7\cos 200°} \approx 142.2°$ because $(-18.0, 14.0)$ is in the second quadrant.
$\beta = 360° - 142.2° + 90° = 307.8°$

5. $\sqrt{(20\cos 125° + 7\cos 200°)^2 + (20\sin 125° + 7\sin 200°)^2}$
≈ 22.8 mi
22.8 mi at 307.8°

6.

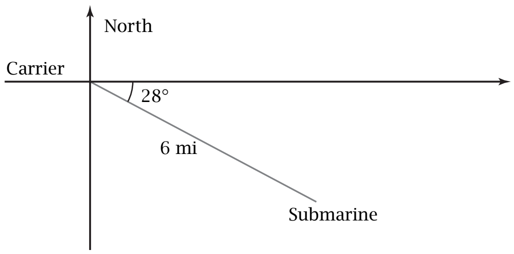


7. This answer is the same as in Problem 5, with different units and the vector of length 7 units translated, which makes no difference to the answer. So the answer is the same (just with different units), 22.8 knots at 307.8°.

8. Answers will vary.

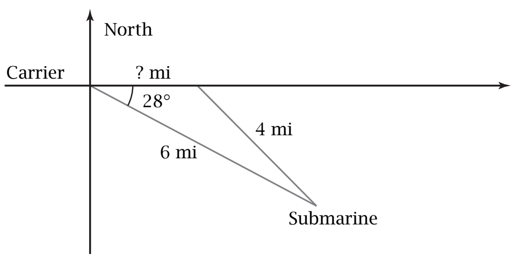
Exploration 6-7a

1. Sketch not to scale.



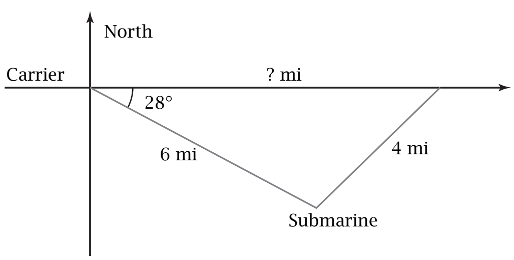
2. (See sketch in Problem 3.)
$4^2 = x^2 + 6^2 - 2 \cdot x \cdot 6\cos 28°$
$x^2 - 12x\cos 28° + 36 - 16 = 0$
$x^2 + (-12\cos 28°)x + 20 = 0$
$x = \dfrac{12\cos 28° \pm \sqrt{(-12\cos 28°)^2 - 4 \cdot 1 \cdot 20}}{2 \cdot 1} \approx 2.5$ mi
(the answer for this problem) or 8.1 mi (the answer for Problem 4)

3. Sketch not to scale.



When drawn to scale, the distance is 2.5 cm.

4. 8.1 mi (See Problem 2.)



5. $\dfrac{\sin\theta}{8.1376\ldots} = \dfrac{\sin 28°}{4}$
$\theta = \sin^{-1}\dfrac{(8.1376\ldots)\sin 28°}{4} \approx 107.2°$ because the angle is obtuse.

6. $\dfrac{1}{2} \cdot 6 \cdot 4\sin\theta \approx 11.5$ mi^2

7. $2^2 = x^2 + 6^2 - 2 \cdot x \cdot 6\cos 28°$
$x^2 - 12x\cos 28° + 36 - 4 = 0$
$x^2 + (-12\cos 28°)x + 32 = 0$
The discriminant is $(-12\cos 28°)^2 - 4 \cdot 1 \cdot 32 \approx -15.7$. So there is no real solution.

8. Answers will vary.

Exploration 6-7b

1. $\theta = \dfrac{360°}{8} = 45°$
$A_{\text{triangle}} = \dfrac{1}{2} \cdot 10 \cdot 10\sin 45° = 25\sqrt{2}$
≈ 35.4 units^2
$A_{\text{octagon}} = 8 \cdot A_{\text{triangle}} = 8 \cdot 25\sqrt{2}$
≈ 282.8 units^2

2. $n \cdot \dfrac{1}{2} \cdot 10 \cdot 10\sin\dfrac{360°}{n} = 50n\sin\dfrac{360°}{n}$

3.

n	**Area**
3	129.9038…
4	200
5	237.7641…
6	259.8076…
7	273.6410…
8	282.8427…
9	289.2544…
10	293.8926…
11	297.3524…
12	300

4. Square: $200\sin 90° = 200 \cdot 1 = 200$ units^2
Dodecagon: $600\sin 30° = 600 \cdot \dfrac{1}{2} = 300$ units^2

5. Answers will vary.

6. More and more digits from the left remain the same.

7. 314.1592…, that is, 100π. The n-gons are approaching a circle with radius 10, whose area would be $\pi \cdot 10^2 = 100\pi$.

8. Answers will vary.