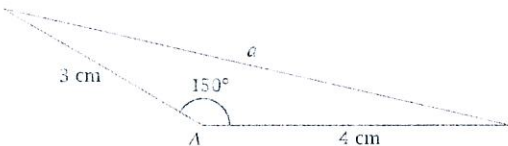
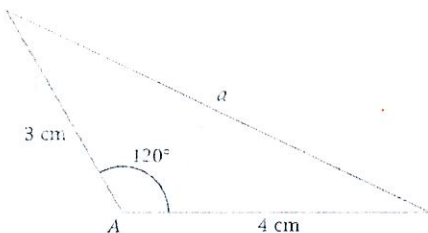
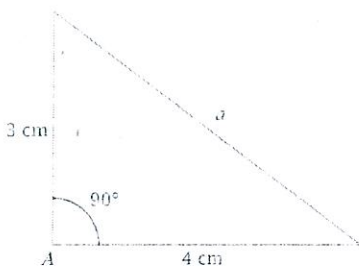
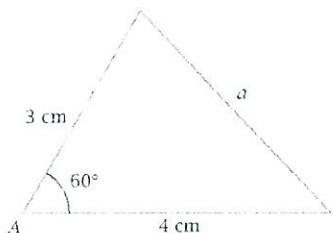
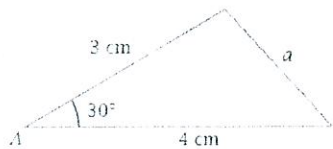


Exploration 6-1a: Introduction to Oblique Triangles

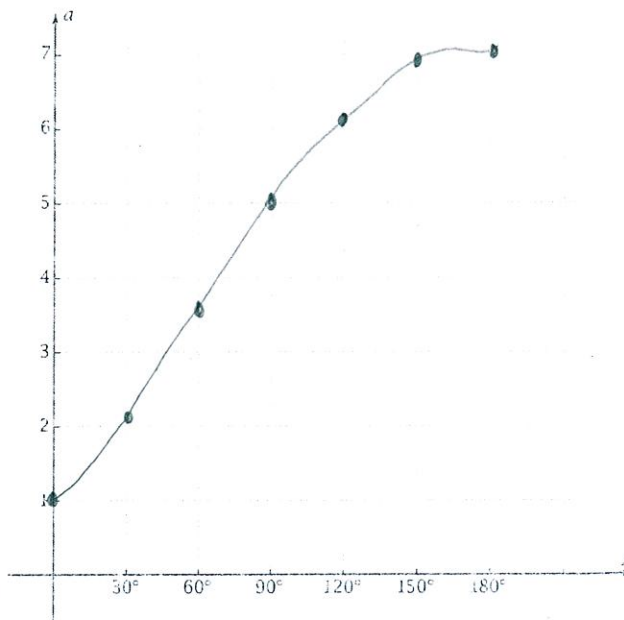
Objective: Given the lengths of two sides of a triangle and the measure of the included angle, measure the third side.



- The figures show five triangles. Each has sides of 3 cm and 4 cm. They differ in the size of the angle included between the two sides. Are the measurements of the lengths of the sides correct as marked on each figure? Yes
- Are the degree measurements correct as shown in each figure? Yes
- Measure the third side, a , of each triangle. Record the results here, correct to one decimal place.

A (degrees)	a (cm)
30°	2.1
60°	3.6
90°	5.0
120°	6.1
150°	6.8

- What would a equal if $\angle A$ were:
 $180^\circ: a = \underline{7}$ $0^\circ: a = \underline{1}$
- Plot the measured values of a as a function of the angle measure, A , from 0° to 180° . Connect the points with a smooth curve.



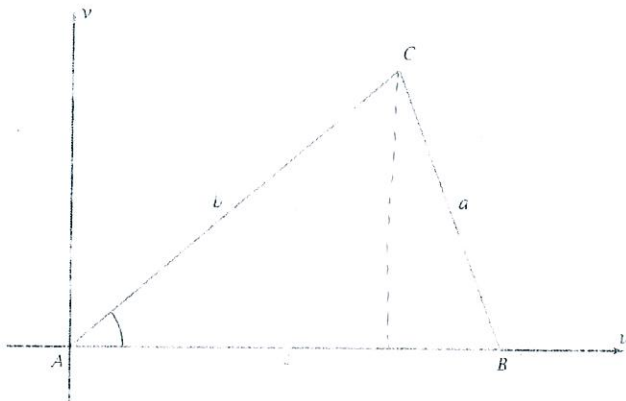
- By the Pythagorean theorem, $a^2 = 3^2 + 4^2$ when A is 90° . If A is less than 90° , you have to subtract something to get the value of a^2 . Consult your textbook to find out what expression must be subtracted.
 $-2bc\cos A$
 $-2(3)(4)\cos A$
 $-24\cos A$
- The answer to Problem 6 is part of the law of cosines. Use the law of cosines to calculate a for each value of angle A in the table of Problem 3.

30°	2.05
60°	3.60
90°	5
120°	6.08
150°	6.77
- What did you learn as a result of doing this Exploration that you did not know before?

Exploration 6-2a: Derivation of the Law of Cosines

Objective: Derive the law of cosines for predicting the third side of a triangle from two sides and the included angle.

The figure shows triangle ABC . Angle A has been placed in standard position in a uv -coordinate system.



1. The sides that include angle A have lengths b and c . Write the coordinates of points B and C using b , c , and functions of angle A .

$$B: (u, v) = (\underline{c}, \underline{0})$$

$$C: (u, v) = (\underline{b \cos A}, \underline{b \sin A})$$

2. Use the distance formula to write the square of the length of the third side, a^2 , in terms of b , c , and functions of angle A .

$$\begin{aligned} a^2 &= (b \cos A - c)^2 + (b \sin A - 0)^2 \\ &= b^2 \cos^2 A - 2bc \cos A + c^2 + b^2 \sin^2 A \end{aligned}$$

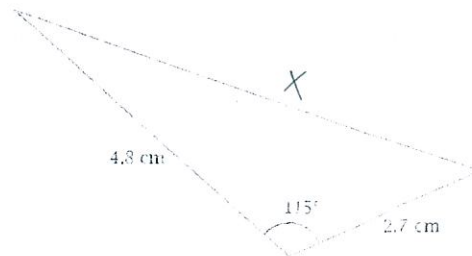
3. Simplify the answer to Problem 2 by expanding the square. Use the Pythagorean property for cosine and sine to simplify the terms containing $\cos^2 A$ and $\sin^2 A$.

$$a^2 = b^2 \cos^2 A - 2bc \cos A + c^2 + b^2 \sin^2 A$$

$$a^2 = b^2 (\cos^2 A + \sin^2 A) - 2bc \cos A + c^2$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

4. The answer to Problem 3 is called the law of cosines. Show that you understand what the law of cosines says by using it to calculate the third side of this triangle.



$$X^2 = 4.8^2 + 2.7^2 - 2(4.8)(2.7)\cos 115^\circ$$

$$X^2 = 41.28$$

$$X = 6.43$$

5. Measure the given sides and the angle of the triangle in Problem 4. Do you agree with the given measurements? Measure the third side. Does it agree with your calculated value?

Yes

6. Describe how the unknown side in the law of cosines is related to the given angle and how the given angle is related to the two given sides, using terms you studied in geometry.

The unknown side is opposite the given angle. The two given sides include the given angle.

7. What have you learned as a result of doing this Exploration that you did not know before?