Warm-up

Use the given point on the terminal side of angle θ to find the value of all 6 trigonometric function indicated.

1) $(-5, -\sqrt{11})$

Find the exact values of the five trigonometric ratios not given.

2) $\csc\theta = -\frac{5}{3}$ and $\cos\theta > 0$

Intro to Trigonometric Identities

EQ: How do I prove trigonometric expressions equivalent?

Oct 6-8:23 AM

Clear your desks

What is an identity?

A statement that is valid for all values of the variables in the domain where the expressions are defined.

What are some identities that we are already familiar with from Algebra?

Oct 6-8:44 AM

Reciprocal Identities

csc x = tan x = sin x =

sec x = cot x = cos x =

Quotient Identities

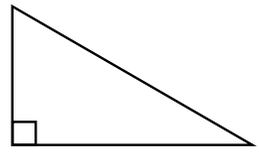
If csc x = 7/4 find sin x

if $\cot x = \frac{2}{5\sqrt{5}}$ and $\sin x = \frac{\sqrt{5}}{3}$ find cos x

Oct 6-9:05 AM

Activity

Pythagorean Identities: Derive the Pythagorean Identity from a right triangle



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$\cos^2\theta + \sin^2\theta = 1$

$\cos^2\theta + \sin^2\theta = 1$

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When simplifying a trig expression, try changing everything to terms of sine and cosine.

Example 1 : Simplify $\cos x \csc x$

$= \cos x \frac{1}{\sin x}$

$= \frac{\cos x}{\sin x}$

$= \cot x$

Example 2: Simplify $\frac{\tan x \cos x}{\sin x}$

$= \tan x \cdot \frac{\cos x}{\sin x}$

$= \frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x} = 1$

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Ex 3: $\frac{\tan x}{\sec x}$

Ex 4: $\cos x - \cos x \sin^2 x$

$= \cos x (1 - \sin^2 x)$

$= \cos x (\cos^2 x)$

$= \cos^3 x$

$\cos^2 x + \sin^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

Ex 5: $\sin x + \cos x \cot x$

$= \sin x + \cos x \frac{\cos x}{\sin x}$

$\frac{\sin x}{\sin x} \sin x + \frac{\cos^2 x}{\sin x}$

$= \frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x}$

$= \frac{1}{\sin x} = \csc x$

Ex 6: $\frac{\sec x - \cos x}{\sec x}$

Ex 7 $\frac{\sin x \cos x}{1 - \sin x} - \frac{1 + \sin x}{\cos x}$

Ex 7. $\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x}$

Ex8. $\frac{1}{1 + \cos x} \cdot \frac{(1 - \cos x)}{(1 - \cos x)} = \frac{1 - \cos x}{1 - \cos^2 x}$

$= \frac{1 - \cos x}{\sin^2 x}$

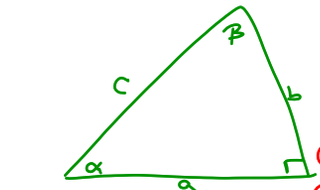
$= \frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x}$

$= \csc^2 x - \cot x \frac{1}{\sin x}$

$= \csc^2 x - \cot x \csc x$

Ex10

$\frac{\cos^2 x}{1 - \sin x}$



$\sin\alpha = \cos\beta$

$\sin x = \cos(90-\alpha)$

$\cos\alpha = \sin\beta$

$\cos\alpha = \sin(90-\alpha)$

$\csc\alpha = \sec\beta$

$\csc\alpha = \sec(90-\alpha)$

$\tan\alpha = \cot\beta$

Sine odd

$\sin(-x) = -\sin(x)$

Even y-axis

$f(x) = f(-x)$

odd origin symmetry rotational symmetry

$f(-a) = -f(a)$

Cosine Even

$\cos(-x) = \cos(x)$

tangent odd

$\tan(-x) = -\tan(x)$

csc odd

$\csc(-x) = -\csc(x)$

Sec Even

$\sec(-x) = \sec(x)$

Cot odd

$\cot(-x) = -\cot(x)$

$\tan\theta = 1.38$ Find $\cot(\theta - \frac{\pi}{2})$

$\tan\theta = \cot(\frac{\pi}{2} - \theta)$

odd → $= \cot(-(\theta - \frac{\pi}{2}))$

$= -\cot(\theta - \frac{\pi}{2})$

$1.38 = -\cot(\theta - \frac{\pi}{2})$

$-1.38 = \cot(\theta - \frac{\pi}{2})$

$\sin\theta = 1.1$ Find $\cos(\theta - \frac{\pi}{2})$

$\sin\theta = \cos(\frac{\pi}{2} - \theta)$

Even → $= \cos(-(\frac{\pi}{2} + \theta))$

$1.1 = \cos(\theta - \frac{\pi}{2})$

Proofs: When in doubt, change everything to sine and cosine. Try to only change one side, not both. Start with the more complicated-looking side.

Example 1: Prove *sin x sec x = tan x*

LHS

$\tan x = \tan x$

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Example 2: Prove $\frac{\csc x}{\cot x} = \sec x$

LHS

$\frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}} =$

$\frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} =$

$\frac{1}{\cos x} =$

$\sec x = \sec x$

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Ex. $\sec x + \sin x = \frac{1 + \sin x \cos x}{\cos x}$

RHS $= \frac{1}{\cos x} + \frac{\sin x \cos x}{\cos x}$

$= \sec x + \sin x$

OR

LHS $\frac{1}{\cos x} + \frac{\sin x \cos x}{\cos x}$

$\frac{1}{\cos x} + \sin x \cos x$

Ex $\cot x + \sec x = \frac{\cos^2 x + \sin x}{\sin x \cos x}$

Example 3: Prove $\cot x + \sec x = \frac{\cos^2 x + \sin x}{\sin x \cos x}$

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Example 1: $\frac{\tan^2 x + 1}{\csc^2 x - 1} = \sec^2 x \tan^2 x$

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Example 2: Prove $\frac{\sec x}{\cot x + \tan x} = \sin x$

Oct 6-9:25 AM

If $\tan\theta = -8$ and $\sin\theta > 0$

Find $\sin\theta + \cos\theta$

$1 + \tan^2\theta = \sec^2\theta$

$1 + (-8)^2 = \sec^2\theta$

$-\sqrt{65} = \sqrt{\sec^2\theta}$

$-\sqrt{65} = \sec\theta$

$\cos\theta = \frac{1}{-\sqrt{65}} = -\frac{\sqrt{65}}{65}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$-8 = \frac{\sin\theta}{-\frac{\sqrt{65}}{65}}$

$\sin\theta = -8\left(-\frac{\sqrt{65}}{65}\right)$

$= \frac{8\sqrt{65}}{65}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$1 + \tan^2\theta = \sec^2\theta$

If $\cot\theta = -3$ and $\cos\theta < 0$
Find $\csc\theta$ and $\tan\theta$

If $\sin x = \frac{1}{6}$ $\cos x > 0$
Find $\cot x + \sec x$

Simplify $\frac{1}{1+\cos x} \cdot \frac{1-\cos x}{1-\cos x}$

$= \frac{1-\cos x}{1-\cos^2 x} = \frac{1-\cos x}{\sin^2 x} = \frac{1}{\sin^2 x} - \frac{\cos x}{\sin^2 x}$

$= \csc^2 x - \frac{\cos x}{\sin x \sin x}$

$= \csc^2 x - \cot x \csc x$

$\frac{\cos^2 x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{\cos^2 x(1+\sin x)}{1-\sin^2 x}$

$= \frac{\cos^2 x (1+\sin x)}{\cos^2 x} = 1+\sin x$

$\frac{4}{\sec x + \tan x} \cdot \frac{\sec x - \tan x}{\sec x - \tan x}$

$= \frac{4(\sec x - \tan x)}{\sec^2 x - \tan^2 x}$

$= \frac{4(\sec x - \tan x)}{(1+\tan^2 x) - \tan^2 x} = 4(\sec x - \tan x)$