What is a determinant?

Determinant - a real number associated with a square matrix. The determinant of a matrix A is denoted by *det A* or |A|.

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A. Determinant of a 2 x 2 matrix

How do I find the determinant of a 2x2?

The determinant of a 2 x 2 matrix is the difference of the products of the elements on the diagonals.

$$\det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

Example:

Ex. Evaluate the determinant of the matrix $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$

$2(4) - 3(-1) = 11$

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Ex #1 Evaluate each determinant

a) $\det\begin{bmatrix} 5 & 8 \\ 9 & 4 \end{bmatrix}$

$= 5(4) - 9(8)$

$= -52$

b) $\det\begin{bmatrix} 3 & -4 \\ 7 & -2 \end{bmatrix}$

$= 3(-2) - (7)(-4)$

$= 22$

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B. Determinant of a 3 x 3 matrix

How do I find the determinant of a 3x3?

Rewrite the first two columns to the right of the determinant. Add the products of the leading diagonals and subtract from this the products of the opposite diagonals.

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \ne \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}\begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$$

$= (aei + bfg + cdh) - (gec + hfa + idb)$

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Example:

Ex. Evaluate the determinant of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & -2 \\ 4 & 1 & 5 \end{bmatrix}\begin{matrix} 1 & 2 \\ 0 & 3 \\ 4 & 1 \end{matrix}$$

$\det(A) =$

$(1\cdot 3\cdot 5 + 2\cdot -2\cdot 4 + (-1)\cdot 0\cdot 1) -$

$(4\cdot 3\cdot -1 + 1(-2)\cdot 1 + 5(0)2)$

$= (15 - 16) - (-12 - 2)$

$= -1 + 14 = \boxed{13}$

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Example:

Ex. Evaluate:

$$\begin{vmatrix} -1 & 3 & 5 \\ 0 & x & 3 \\ -2 & 2 & x \end{vmatrix} = x - 4$$

$(-1\cdot x\cdot x + 3\cdot 3\cdot -2 + 5(0)2) -$

$(-2\cdot x\cdot 5 + 2\cdot 3\cdot 1 + x\cdot 0\cdot 3) = x - 4$

$(-x^2 - 18) - (-10x - 6) = x - 4$

$-x^2 - 18 + 10x + 6 = x - 4$

$-x + 4 \quad -x + 4$

$-x^2 + 9x - 8 = 0$

$-(x^2 - 9x + 8) = 0$

$-(x-8)(x-1) = 0$

$x = 8, 1$

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C. Applications of Determinants

How can I use determinants?

The determinant of a matrix can be used to find the area of a triangle.

If (x_1, y_1), (x_2, y_2), and (x_3, y_3) are vertices of a triangle, the area of the triangle is:

$$\text{Area} = \pm \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

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Example:

Ex. Find the area of the triangle given the points (-2, 1), (4, 7), (9, 8).

$$A = \pm\frac{1}{2}\begin{vmatrix} -2 & 1 & 1 \\ 4 & 7 & 1 \\ 9 & 8 & 1 \end{vmatrix}$$

$$= \pm\frac{1}{2}\left[(-2\cdot 7\cdot 1 + 1\cdot 1\cdot 9 + 1\cdot 4\cdot 8) - (9\cdot 7\cdot 1 + 8\cdot 1\cdot -2 + 1\cdot 4\cdot 1)\right.$$

$$= \pm\frac{1}{2}(27 - 51)$$

$$= \pm\frac{1}{2}(-24)$$

$$= 12$$

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Vocabulary:

What is an identity matrix?

Identity matrix - a square matrix with 1's along the leading diagonal and 0's elsewhere.

- An identity matrix is denoted using *I*.
- *AI = IA = A*.

What is an inverse?

Inverses - the square matrices *A* and *B* are inverses of each other if their product (in both orders) is equal to an identity matrix.

i.e. *AB=I* and *BA=I*.

- Matrix *A* has an inverse iff det *A* ≠ 0
- The inverse of matrix *A* is denoted as A^{-1}.

Mar 23-1:27 PM

A. Find the inverse of a 2 x 2 matrix

How do I find the inverse of a 2x2?

Given the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and assuming $ad - cb \neq 0$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

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Example: Find the inverse of each matrix and verify the matrices are inverses.

$$A = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 6 & 2 \\ 9 & 3 \end{bmatrix}$$

$\det(A) = 2 + 3 = 5$ ✓

$$A^{-1} = \frac{1}{5}\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

$A \cdot A^{-1} =$

$$\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{5} & -\frac{3}{5} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{5} + \frac{3}{5} & -\frac{3}{5} + \frac{3}{5} \\ -\frac{2}{5} + \frac{2}{5} & +\frac{3}{5} + \frac{2}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\det B = 6(3) - 9(2)$

$= 0$

No inverse

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B. Find the inverse of a 3 x 3 matrix

How do I find the inverse of a 3x3?

Use a calculator!

Ex. Find the inverse of $\begin{bmatrix} 4 & 0 & -1 \\ 6 & -2 & 0 \\ 3 & 1 & -4 \end{bmatrix}$

Verify they are inverses.

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