Applications of Trig Functions

Graph the following trig functions with your graphing calculator, what is the period?

$y = 2\sin\left(\frac{2\pi}{3}[x-1]\right)+3$ $\qquad$ $y = 5\cos\left(\frac{2\pi}{5}[x-4]\right)+1$

In order obtain a period given, use $y = \sin\left(\frac{2\pi}{p}x\right)$ and $y = \cos\left(\frac{2\pi}{p}x\right)$

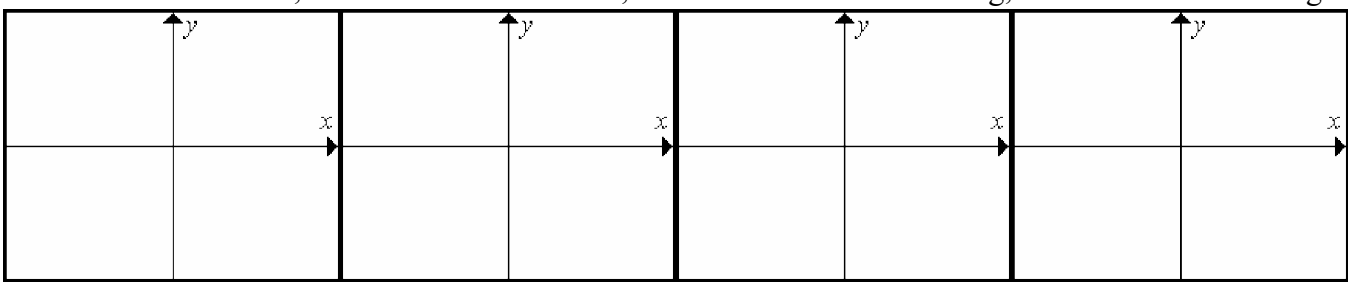
Write an equation for a cosine function with the following properties:

a)
- Amplitude: 3
- Period: 2
- Phase shift: -4
- Vertical displacement: 6

b)
- Maximum: 25
- Minimum: 13
- Period: 6
- Phase shift: 3

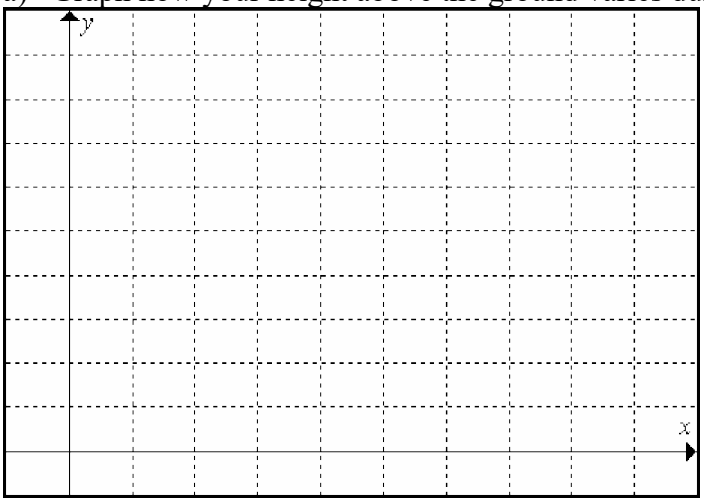
Some convenient methods for creating a function to match applications:

Starts at maximum, starts at minimum, starts at centre and rising, starts at centre/falling



A Ferris wheel has a radius of 42 m. Its centre is 43 m above the ground. It rotates once every 50 s. Suppose you get on at the bottom at $t = 0$.

a) Graph how your height above the ground varies during the first two cycles.

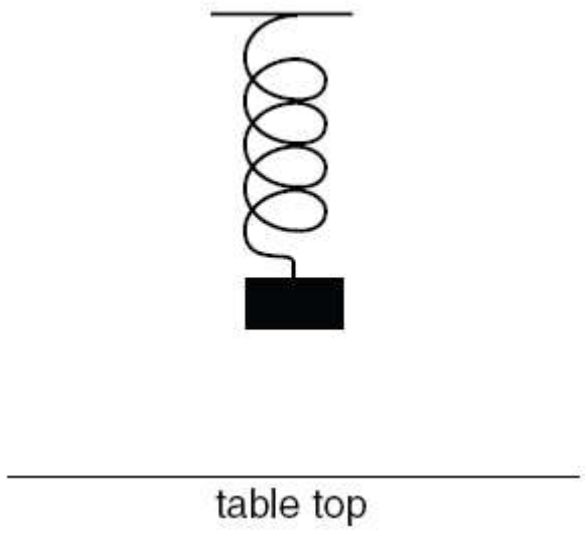


b) Write an equation that expresses your height as a function of the elapsed time.

c) Estimate your height above the ground after 65 s.

d) Estimate one of the times when your height is 25 m above the ground.

A certain mass is rests at 0.5m above a table top. The mass is pulled down 0.4m and released at time t = 0, rising to a maximum height of 0.9m above the table. It takes 1.2s for the mass to return to the low position each time. Determine an equation to model to height of the spring above the table.



At a seaport, the water has a maximum depth of 15 m at 7:00 a.m. The minimum depth of 5 m occurs 1:30pm. Assume the relation between the depth of the water and time is a sinusoidal function.

a) Write an equation for the depth, *h* meters, of the water at any time, *t* hours.

b) Estimate the depth at 11:00 a.m.

c) A cargo ship's hull floats at a depth of 8m. How long can it be in port to load its cargo?

An ant gets stuck in your car tire as you drive through a creek 0.3m deep. If your tire is 0.5m tall and rotates once every 0.5sec, how long will the ant have to hold its breath for?

Practice

1) The height of the tide at a dock can be modeled by the equation

$$h = 4.5\sin(0.5t - 1.5) + 7.3$$

where h is the height of the tide in meters at time t (using a 24 hour clock)

a) What is the maximum height of the tide?
b) What is the minimum height of the tide?
c) What is the median height of the tide?
d) What is the period of this function?
e) What is the height of the tide at a) 8:00 am b) 4 pm

2) In a seaport, the function $d(t) = 2.6\sin 0.25(t-5) + 3.3$ can be used to estimate the depth of water, d meters, at time t hours after midnight. Estimate the number of hours in the 24-hour interval starting at $t = 0$ when the depth is at least 3.5m

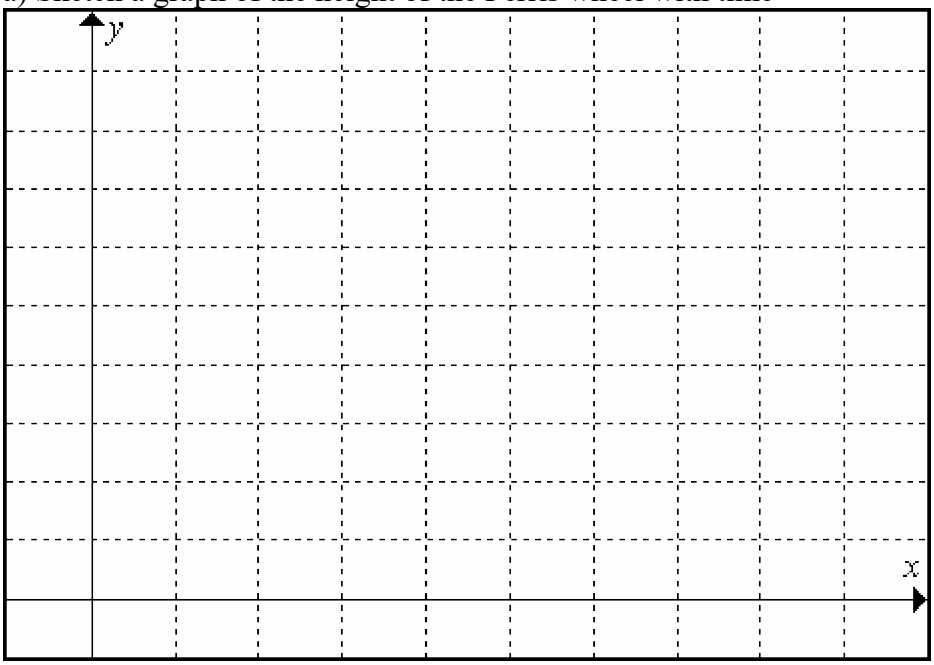
3) At seaport, the water has a maximum depth of 18m at 3:00am. After this maximum depth, the first minimum depth of 4m occurs at 9:30am. Assume that the relation between the depth , h meters, and the time t hours, is a sinusoidal function. Determine an equation for h at any time t.

4) A wheel with diameter 10cm is rolling along the ground. Point P is on the edge of the wheel on the ground at time $t = 0$ seconds. Find an equation for the height of the point P above the ground at time t seconds, if the wheel rotates once every 12 seconds.

5) Lenny and Carl get on a Ferris wheel half way between the bottom and the top of the Ferris wheel, which is located 14m above the ground and are rising when they start. They complete four revolutions lasting a total of 16mins. At the highest point on the ride, Lenny and Carl are 24m above the ground. Determine a sinusoidal function for this problem letting h represent their height in meters above the ground and t time in minutes.

6) The radius of a Ferris wheel is 12 m, and the wheel rotates once every 40 seconds. A person sits 14 m above the ground and is falling when the wheel starts to rotate. The lowest height is 2 m above the ground.

a) Sketch a graph of the height of the Ferris wheel with time



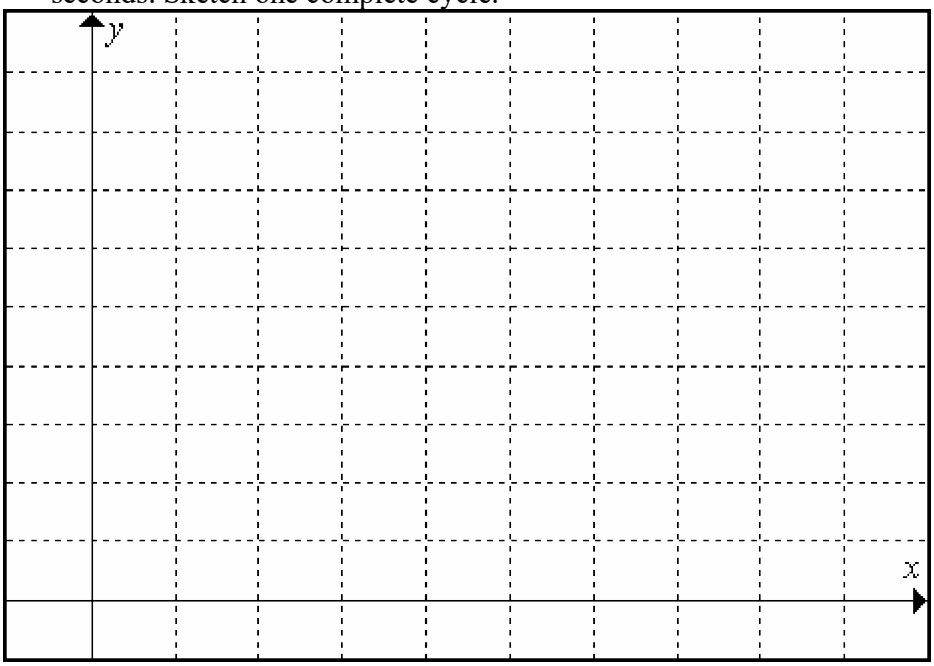
b) Determine an equation for the height of the person at time t

c) How high is the person 25 seconds after the wheel starts turning?

d) What is the first time the person is 6 m above the ground?

7) A Ferris wheel has a radius of 10m and the maximum height the chair reaches is 22m. The wheel takes 90 seconds to complete one revolution and the rider starts at the bottom.

a) Sketch a graph that represents the height h in meters, of the bottom chair, as a function of time t, in seconds. Sketch one complete cycle.



b) Write an equation in terms of cosine, that expresses the height h of the bottom chair, as a function of time in seconds. That is $h = a\cos\left(b\left[x-c\right]\right)+d$.

c) What is the lowest height that any chair is above the ground?

d) Estimate the height the chair is above the ground after 20 seconds.

e) Estimate both times when the chair is 19m above the ground.

8) The diameter of a Ferris wheel is 76 meters and the maximum height of the Ferris wheel is 80m. If the wheel rotates every 3 minutes and riders on a cart start at the lowest point,

a) Determine a sinusoidal equation to model the situation.

b) How high is the cart 5 minutes after the wheel starts rotating

c) How many seconds after the wheel starts rotating is the cart 50m above the ground for the first time?

9) A Ferris wheel has a diameter of 30m, with the center 18m above the ground. It makes one revolution every 60s. If the rider gets on at the top,

a) Find a cosine equation to model this situation

b) What is the height of the rider at 52 seconds?

c) At what time(s) is the rider at 20m high?

10) A water wheel on a paddleboat has a radius of 1m. The wheel rotates once every 1.46 seconds and the bottom 0.3m of the wheel is submerged in the water.

a) Determine a sinusoidal equation, starting from a point at the top of the wheel

b) How long is the point submerged?

11) The average temperature for Regina is hottest at $27°C$ on July 12^{th}, and coolest at $-16°C$ on January 10^{th} (assume 29 days in February).

a) Write a sinusoidal equation to model the temperature

b) What is the expected average temperature for March 15^{th}?

c) How many days will the average temperature be over $20°C$?

HW: MC – 2, 10, 20, 30, 39, 49, 70 WR – 7