Unit 7 - Probability

Lesson 3: Combinations

Essential Question: How do I count dependent outcomes without order?

MCC9-12.S.CP.9 Use permutations and combinations to compute probabilities of compound events and solve problems

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Permutation and Combination

Permutation : Permutation means *arrangement* of things. The word *arrangement* is used if the order of things *is considered*.

Think phone numbers--order is important!

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Combination: Combination means *selection* of things. The word *selection* is used when the order of things has *no importance*.

Think selecting 5 basketball players to start Saturday's game--selection order is NOT important!

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Permutation

'n' different things taken 'r' at a time:

$$ {}_nP_r = \frac{n!}{(n-r)!} $$

0! is defined to be 1

***Order Matters!!!

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Combinations

'n' different things, taken 'r' at a time:

$n \geq r$

$$ {}_nC_r = \frac{n!}{r!(n-r)!} $$

*** order does NOT matter!!!***

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1. ${}_8C_3 = \frac{8!}{3!(8-3)!} = \frac{8!}{3!5!} = \frac{8 \cdot 7 \cdot 6 \cdot 5!}{3 \cdot 2 \cdot 1 \cdot 5!} = 56$

2. ${}_{10}C_4 = \frac{10!}{4!6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 6!} = 210$

3. ${}_{15}C_5 = \frac{15!}{5!10!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 10!} = 3003$

4. ${}_{15}C_{10} = \frac{15!}{10!5!} = \frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 10!} = 3003$

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Symmetry of Combinations

$$_nC_r = {}_nC_{n-r}$$

Example: Compare $_8C_2$ to $_8C_6$

$\frac{8!}{6!2!} = \frac{8 \cdot 7 \cdot 6!}{6! \cdot 2} = 28$ $\frac{8!}{2!6!} = \frac{8 \cdot 7 \cdot 6!}{2 \cdot 1 \cdot 6!} = 28$

Why does this happen?

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What's the difference?

$$_nP_r = \frac{n!}{(n-r)!} \qquad _nC_r = \frac{n!}{r!(n-r)!}$$

What causes that difference?

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Ex 1: How many combinations of 3 letters can you come up with using the letters A⇨F?

$$_6C_3 = \frac{6!}{3!(6-3)!} = \frac{6!}{3!3!} = \frac{6\cdot5\cdot4\cdot3!}{3!\cdot3\cdot2\cdot1} = 20$$

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Ex 2: For your school pictures, you can choose 4 backgrounds from a list of 10. How many combinations of backdrops are possible?

$$_{10}C_4 = \frac{10!}{4!6!} = \frac{10\cdot9\cdot8\cdot7\cdot6!}{4\cdot3\cdot2\cdot1\cdot6!} = 210$$

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Ex 3: The student council has 16 members, including 4 freshmen. A committee of four is created. What is the probability that all four members are freshmen?

$_{16}C_4 = \frac{16!}{12!4!} = \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12!}{12! \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 1820$ total

$_4C_4 = \frac{4!}{4!0!} = 1$

$\frac{1}{1820} = .0549\%$

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Ex 4. Your math teacher wants to create a group that has 5 students in it. If your class has 5 boys and 14 girls, what is the probability that the group will include only boys? What is the probability that it will include only girls?

$_{19}C_5 = \frac{19!}{5!14!} = \frac{19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 14!} = 11628$

$_5C_5 = 1$

boys: $\frac{1}{11628}$

$_{14}C_5 = \frac{14!}{5!9!} = \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 9!} = 2002$

girls: $\frac{2002}{11628} = 17.22\%$

Challenge: What is the probability that the group will have 3 girls and 2 boys?

$_{14}C_3 \cdot {}_5C_2 = \frac{14!}{3!11!} \cdot \frac{5!}{2!3!} = \frac{14 \cdot 13 \cdot 12 \cdot 11! \cdot 5 \cdot 4 \cdot 3!}{3 \cdot 2 \cdot 11! \cdot 2 \cdot 1 \cdot 3!}$

$= \frac{3640}{11628} = 31.3\%$

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$${}_nC_r = \binom{n}{r}$$

$$\binom{8}{4} = {}_8C_4 = \frac{8!}{4!4!} = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4!}{4\cdot 3\cdot 2\cdot 1\cdot 4!} = 70$$

Dec 3-9:14 AM