Unit 7 - Permutations and Combinations
A Study in Probability & Statistics

Lesson 2: Permutations

Essential Question: How do I count dependent outcomes with order?

Feb 5-10:50 AM

Counting Principle Warm-up:

- A certain state license plate is made by 4 letters followed by 2 numbers. How many license plates are possible?

$\underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\ \underset{L}{\underline{26}}\ \underset{\#}{\underline{10}}\ \underset{\#}{\underline{10}} = 45{,}697{,}600$ license plates *

- How many license plates are possible if you cannot repeat letters or digits?

$\underline{26}\ \underline{25}\ \underline{24}\ \underline{23}\ \underline{10}\ \underline{9} = 32{,}292{,}000$

Aug 17-8:14 AM

Permutation and Combination

Permutation : Permutation means *arrangement* of things. The word *arrangement* is used if the order of things *is considered*.

Think phone numbers--order is important!

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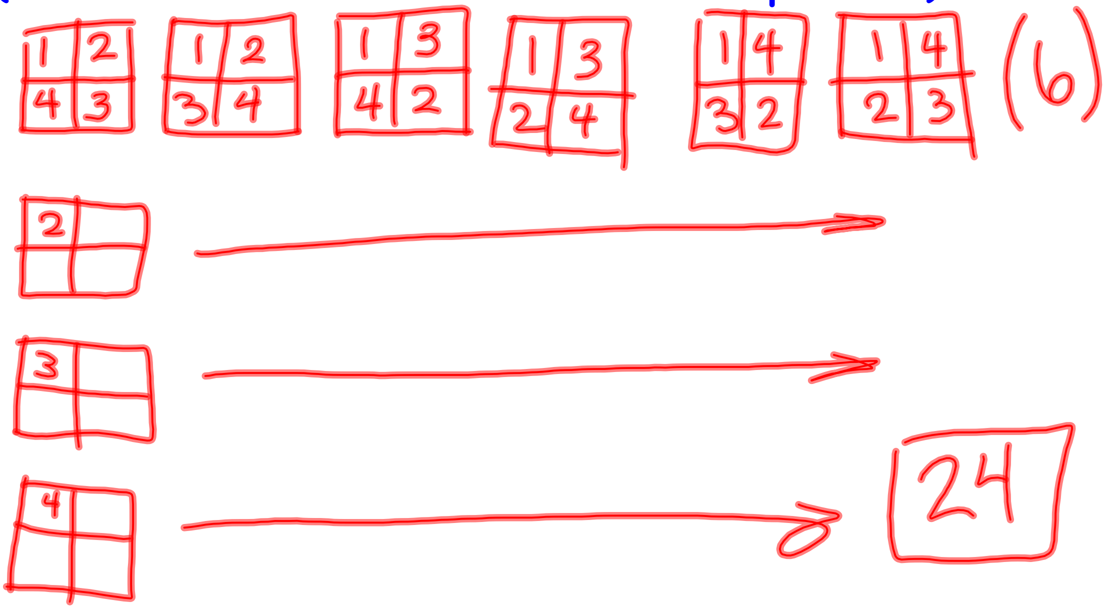
Combination: Combination means *selection* of things. The word *selection* is used when the order of things has *no importance*.

Think selecting 5 basketball players to start Saturday's game--selection order is NOT important!

Feb 5-11:27 AM

Introduction: If you have 4 people playing 4-square, how many ways can they be arranged? (Hint: Write a list of all the options)



Aug 17-8:06 AM

n Factorial: The product of positive integers from 1 to n is denoted by n!.

$$n! = n(n-1)\,(n-2)\ \ldots\ldots\ldots\ldots\ldots\ldots.3 \times 2 \times 1.$$

Ex. $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

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$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40{,}320$

$$\frac{10!}{7!} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = \frac{10 \cdot 9 \cdot 8 \cdot \cancel{7!}}{\cancel{7!}} = 720$$

$$\frac{41!23!}{42!20!} = \frac{\cancel{41!}\, 23 \cdot 22 \cdot 21 \cdot \cancel{20!}}{42 \cdot \cancel{41!}\, \cancel{20!}} = \frac{23 \cdot 22 \cdot 21}{42} = 253$$

$$\frac{\cancel{n!}(n+3)!}{(n+1)!\cancel{n!}} = \frac{(n+3)(n+2)(n+1)!}{(n+1)!} = (n+3)(n+2) = n^2+5n+6$$

Dec 2-8:05 AM

Permutation

'n' different things taken 'r' at a time:

$n \geq r$

$${}_nP_r = \frac{n!}{(n-r)!}$$

0! is defined to be 1

***Order Matters!!!

Feb 5-1:03 PM

1. ${}_{10}P_6 = \frac{10!}{(10-6)!} = \frac{10!}{4!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot \cancel{4!}}{\cancel{4!}} = 151{,}200$

2. ${}_{18}P_{15} = \frac{18!}{(18-15)!} = \frac{18!}{3!} = 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 1.067 \times 10^{15}$

3. ${}_{7}P_{7} = \frac{7!}{(7-7)!} = \frac{7!}{0!} = \frac{7!}{1} = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

4. ${}_{n}P_{n-3} = \frac{n!}{(n-(n-3))!} = \frac{n!}{3!} = n(n-1)(n-2)\ldots(6)(5)(4)$

Dec 2-8:07 AM

Ex 1: There are now 4 people running for school office. The person with the most votes will get president, the person with the second highest will get vice-president, the third person will be treasurer, and the fourth person will be secretary. How many ways can these four people be arranged?

$${}_4P_4 = \underset{P}{4} \cdot \underset{VP}{3} \cdot \underset{T}{2} \cdot \underset{S}{1} = 24$$

$$\frac{4!}{(4-4)!} = \frac{4!}{0!} = 4! = 4 \cdot 3 \cdot 2 \cdot 1$$

Aug 17-8:19 AM

Ex 2: There are 8 people running in a race. In how many ways can the top three runners be decided for gold, silver, and bronze medals?

$${}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = \frac{8\cdot 7\cdot 6\cdot 5!}{5!} = 336$$

$$\frac{8}{G}\cdot\frac{7}{S}\cdot\frac{6}{B} = 336$$

medaling arrangements

Aug 17-8:20 AM

Ex. 3: How many different signals can be made by 5 flags from 8 flags of different colors?

$${}_8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot \cancel{3!}}{\cancel{3!}} = 6720$$

signals

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Ex. 4: How many words (not necessarily logical) can be made by using the letters of the word "WILDCAT" taken all at a time?

$$_7P_7 = \frac{7!}{(7-7)!} = \frac{7!}{0!} = 7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$= 5040 \text{ words}$$

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Ex. 5: How many words (not necessarily logical) can be made by using the letters of the word "WHEELER" taken all at a time?

$$\frac{7!}{3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{\cancel{3!}} = 840$$

How many different words (not necessarily logical) can be made out of 5 of the letters?

$$_7P_5 \ \frac{7!}{(7-5)!\,3!} = \frac{7!}{2!\,3!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{2 \cdot 1 \cdot \cancel{3!}} = 420$$

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Ex. 6: How many words (not necessarily logical) can be made by using the letters of the word "CALCULUS" taken all at a time?

2C's 2L's 2U's

$$\frac{8!}{2!2!2!} = \frac{\cancel{8}\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{\cancel{2}\cdot1\cdot\cancel{2}\cdot1\cdot\cancel{2}\cdot1} = 7! = 5040$$

How many different words (not necessarily logical) can be made by using 6 of the letters?

$${}_8P_6 = \frac{8!}{(8-6)!2!2!2!} = \frac{\cancel{8}\cdot7\cdot6\cdot5\cdot4\cdot3\cdot\cancel{2\cdot1}}{\cancel{2}\cdot1\cdot\cancel{2}\cdot1\cdot\cancel{2}\cdot1\cdot\cancel{2\cdot1}} = 2520$$

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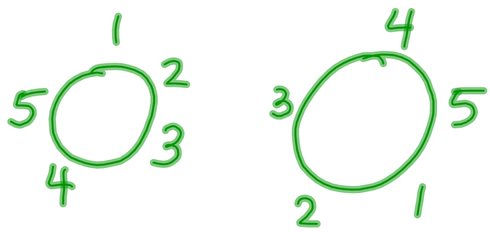
Ex. 7: A group of 8 is going to be seated at a circular table. How many different arrangements can be made?



$$\frac{8!}{8} = 7! = 5040$$

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