

# Unit 7: Probability

## Lesson 5: Discrete Probability Distributions and Expected Value

EQ: How do we use probability to help make decisions?

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### Vocabulary

Probability Distribution - A table listing the probabilities of a certain event.

Probability Histogram - A histogram where the dependent values are the probability of a certain discrete event. The sums of the probability distribution should add to 1.

Expected Value - the weighted average of all possible outcomes.

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### 3 point, 1 point game

- 1) Player A has numbers 5 and 8 and Player B has all the other numbers.
- 2) Each player starts with 10 points.
- 3) Each time a 5 or an 8 is rolled, player A takes 3 points from player B and adds them to his or her own points.
- 4) Each time another number is rolled, player B takes 1 point from player A and adds them to his or her own points.
- 5) At the end of 10 rolls, the player with the most points wins.

Was the game fair? Why or why not?

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Die 1	Die 2	Sum	Die 1	Die 2	Sum
1	1	<del>2</del>	6	1	<del>7</del>
1	2	<del>3</del>	6	2	<del>8</del>
1	3	<del>4</del>	6	3	<del>9</del>
1	4	<del>5</del>	6	4	<del>10</del>
1	5	<del>6</del>	6	5	<del>11</del>
1	6	<del>7</del>	6	6	<del>12</del>
2	1	<del>3</del>	5	1	<del>6</del>
2	2	<del>4</del>	5	2	<del>7</del>
2	3	<del>5</del>	5	3	<del>8</del>
2	4	<del>6</del>	5	4	<del>9</del>
2	5	<del>7</del>	5	5	<del>10</del>
2	6	<del>8</del>	5	6	<del>11</del>
3	1	<del>4</del>	4	1	<del>5</del>
3	2	<del>5</del>	4	2	<del>6</del>
3	3	<del>6</del>	4	3	<del>7</del>
3	4	<del>7</del>	4	4	<del>8</del>
3	5	<del>8</del>	4	5	<del>9</del>
3	6	<del>9</del>	4	6	<del>10</del>
4	1	<del>5</del>	3	1	<del>4</del>
4	2	<del>6</del>	3	2	<del>5</del>
4	3	<del>7</del>	3	3	<del>6</del>
4	4	<del>8</del>	3	4	<del>7</del>
4	5	<del>9</del>	3	5	<del>8</del>
4	6	<del>10</del>	3	6	<del>9</del>
5	1	<del>6</del>	2	1	<del>3</del>
5	2	<del>7</del>	2	2	<del>4</del>
5	3	<del>8</del>	2	3	<del>5</del>
5	4	<del>9</del>	2	4	<del>6</del>
5	5	<del>10</del>	2	5	<del>7</del>
5	6	<del>11</del>	2	6	<del>8</del>
6	1	<del>7</del>	1	1	<del>2</del>
6	2	<del>8</del>	1	2	<del>3</del>
6	3	<del>9</del>	1	3	<del>4</del>
6	4	<del>10</del>	1	4	<del>5</del>
6	5	<del>11</del>	1	5	<del>6</del>
6	6	<del>12</del>	1	6	<del>7</del>

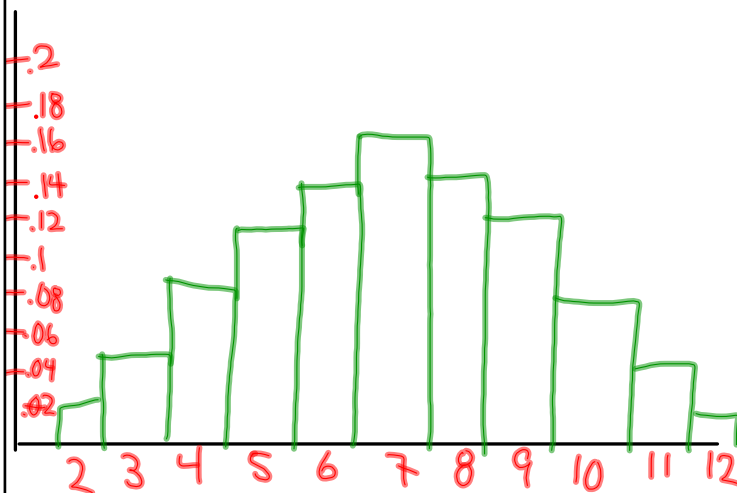
2-1  
 3-2      10-3  
 4-3      11-2  
 5-4      12-1  
 6-5  
 7-6  
 8-5  
 9-4

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Let's draw the probability distribution of the game to see if this helps clarify.

2	3	4	5	6	7	8	9	10	11	12		
$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$		

.027  $\frac{1}{18}$   $\frac{1}{12}$   $\frac{1}{9}$   $\frac{1}{6}$   $\frac{1}{9}$   $\frac{1}{12}$   $\frac{1}{18}$  .027  
 .055 .083 .111 .138 .167 .138 .111 .083 .055



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### What is the value of the event?

$$EV = (\text{prob}_1)(\text{value}_1) + (\text{prob}_2)(\text{value}_2) + \dots$$

When we make our chart, the SUM of the probabilities always equals ONE.

This is simply a **WEIGHTED AVERAGE**.

$$\frac{1(2) + 2(3) + 3(4) + 4(5) \dots}{36}$$

$$\frac{1}{36}(2) + \frac{2}{36}(3) + \frac{3}{36}(4) + \frac{4}{36}(5) \dots$$

✓  
prob.

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### 3 point, 1 point game

A game like this can be simplified to whether the expected value of each turn is equal.

- 1) Each time a 5 or an 8 is rolled, player A takes 3 points from player B and adds them to his or her own points.
- 2) Each time another number is rolled, player B takes 1 point from player A and adds them to his or her own points.

Find the expected value of 1 turn of the game. *9 ways 5 or 8, 27 ways everything else*

Player A	Win	Lose
Probability	$\frac{9}{36} = \frac{1}{4}$	$\frac{27}{36} = \frac{3}{4}$
Value	+3	-1

Player B	Win	Lose
Probability	$\frac{27}{36} = \frac{3}{4}$	$\frac{9}{36} = \frac{1}{4}$
Value	+1	-3

Who do you expect to win?

$$EV_A = \frac{1}{4}(3) + \frac{3}{4}(-1) = 0$$

$$EV_B = \frac{3}{4}(1) + \frac{1}{4}(-3) = 0$$

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What is the average of the following numbers?

5, 5, 10, 10, 10, 10, 20, 25, 30, 30

A game spinner is created such that it follows the table in landing on values. What do you expect to be the average value of 10 spins? 100 spins?

Value	5	10	20	25	30
Probability	20%	40%	10%	10%	20%

$$.2(5) + .4(10) + .1(20) + .1(25) + .2(30) = 15.5$$

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Ex 1: Card Game:

	Player A	Player B
Cards:	1, 2, 3	1, 2, 3
Sum is even:	+4 pts	-2 pts
Sum is odd:	-2 pts	+4 pts

What are the possible outcomes?

1 1

1 2

1 3

2 1

2 2

2 3

3 1

3 2

3 3

Player A

Outcome	Even	Odd
Value	4	-2
Probability	$\frac{5}{9}$	$\frac{4}{9}$

$$EV = \frac{5}{9}(4) + \frac{4}{9}(-2) = \frac{12}{9} = \frac{4}{3}$$

Player B

Outcome	Even	Odd
Value	-2	4
Probability	$\frac{5}{9}$	$\frac{4}{9}$

$$EV = \frac{5}{9}(-2) + \frac{4}{9}(4) = \frac{6}{9} = \frac{2}{3}$$

For Player A, draw the probability distribution.

For Player A, draw the probability histogram.

For both players, find the expected values.

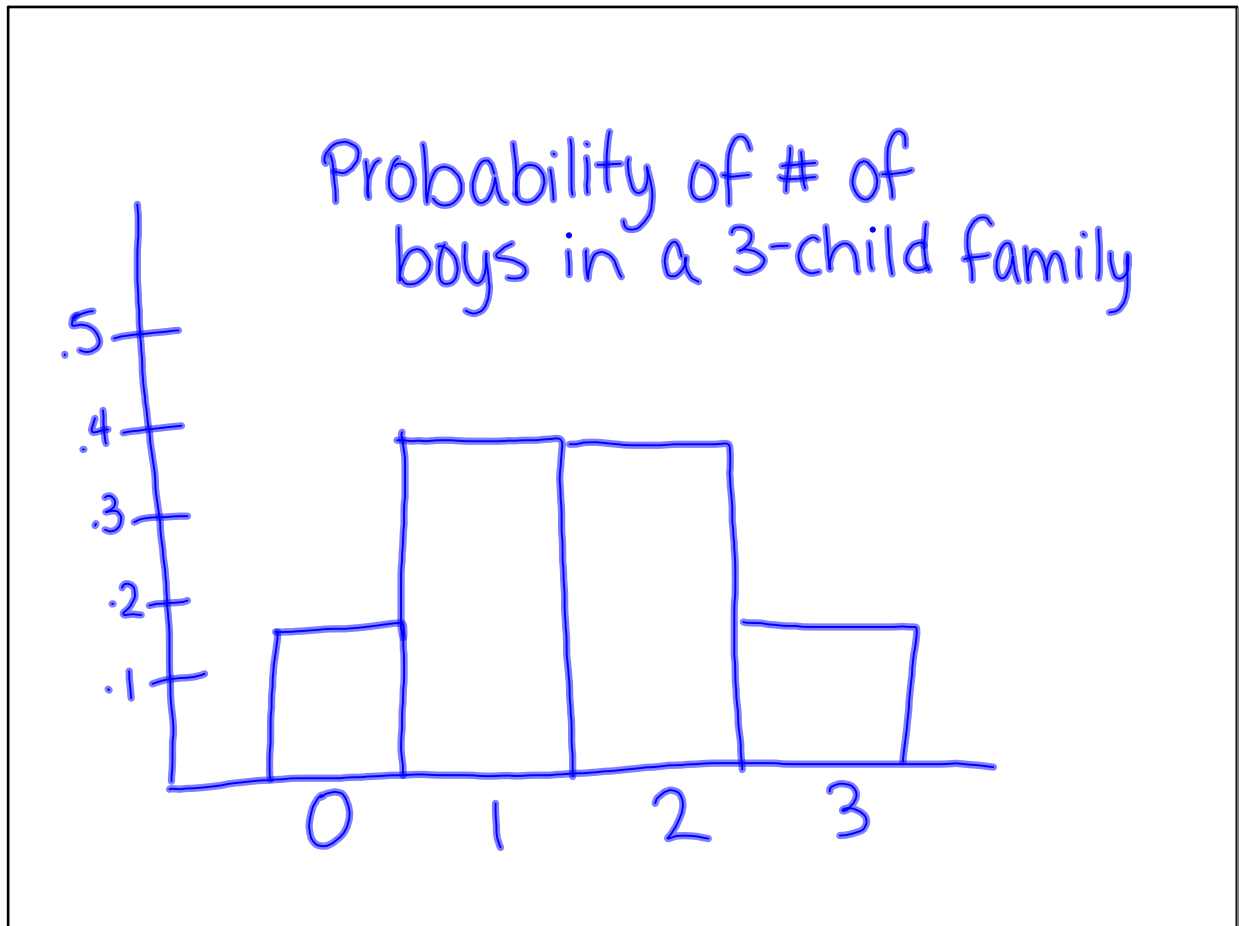
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Let X be the number of boys in a three-child family.

Outcome	0	1	2	3
P(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8} =$

G	G	G	B B B
G	G	B	B B G
G	B	G	B G B
B	G	G	G B B



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G	80	85	90	95	100
P(G)	.1	.4	.2	.2	.1

$$.1(80) + .4(85) + .2(90) + .2(95) + .1(100)$$
$$= 89$$

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