CCGPS Precalculus
Unit 7 Review - Probability

Name:_____

Combinations and Permutations

1. ${}_{12}P_4 = 11,880$

2. ${}_8P_5 = 6,720$

3. ${}_{14}C_{11} = 364$

4. $\binom{n+4}{n+2} = \frac{n^2+7n+12}{2}$

5. You are ordering a pizza. The three-topping pizza costs \$8. If there are 18 toppings to choose from, 4 different crusts, and 2 different sauces, how many different 3-topping pizzas can you order?

6528 pizzas

6. A radio station is playing 10 different songs. If 2 of the songs are going to be played twice, and one of the songs is going to be played 4 times, how many ways can the station fill 15 song slots?

≈1362,608,000 orders

7. Triangles are often labeled by placing a different letter at each vertex. In how many different ways could a given scalene triangle be labeled using any of the 26 letters of the alphabet?

15,600 ways

8. You are picking a hand of 5 cards from a standard deck of playing cards. What is the probability that your hand has 2 10s and 3 Jacks? $\frac{1}{108290} = .0009\%$

9. At South High School, 55 students entered an essay contest. From these students, 10 are selected as finalists. How many different ten-student rankings could be made from the 55 entrants?

1.06×10^{17} ways

Expected Value

1. A student pays \$2 to play the following game. He tossed three coins. If he gets exactly two heads he wins \$7. If he gets exactly one head he wins \$5. Otherwise, he loses his money. On average, how much should he win or lose per play of the game?

wins \$2.50

2. At Tucson Raceway Park, your horse, Stick-in-the-mud has a probably of $\frac{1}{20}$ of coming in first place, a probability of $\frac{1}{10}$ of coming in second, and a probability of $\frac{1}{4}$ of coming in third. First place wins \$4500, second place \$3500, and third place \$1500. It costs you \$1000 to enter the race. What is the expected value of the race to you? Is it worthwhile for you to enter the race? Explain.

\$950

no, you would expect to lose \$50 on average

3. A social club has a drawing every Friday night. The probability of winning the first prize of \$100 is 0.002. The probability of winning the second prize of \$80 is 0.01. How much should the club charge for tickets to enter the drawing so that the club breaks even?

\$1 per ticket

4. You plan to invest in a certain project. There is a 35% chance that you will lose $30,000, a 40% chance that you will break even, and a 25% chance that you will make $55,000. What is the expected value in this problem, and what does it mean in terms of your investment?

$3250, you should invest

Binomial Distribution

1. Find P(5) if n = 12 and p = .40 (show setup)

22.7%

2. Find P(X < 7) if n = 20 and p = .85

.000046%

3. Find P(X ≥ 9) if n = 25 and p = .35

53.32%

4. Find P(3 < X < 6) if n = 10 and p = .42

46.49%

5. Find P(3 ≤ X ≤ 6) if n = 10 and p = .42

79.17%

6. If a dark-haired mother and father have a particular combination of genes, each of their babies have a ¼ probability of having light hair.
   a. What is the probability of any one baby having dark hair? 3/4
   b. If they have 3 babies, calculate P(0), P(1), P(2), and P(3), the probabilities of having 0, 1, 2, and 3 dark-haired babies. 1.56%, 14.06%, 42.19%, 42.19%
   c. Plot the graph of this probability distribution. Does this distribution have a special name?

left skew

7. Statistics show that about 8% of all males are color-blind. Supposed 20 males are selected at random.
   a. Find the probability that 2 of the males are color-blind. 27.11%
   b. Find the probability that more than 5 of the males are color-blind. .38%
   c. Find the probability that less than 3 of the males are color-blind. 78.79%
   d. Find the probability that between 4 and 7 (inclusive) of the males are color-blind. 7.05%

8. Large tractor-trailer trucks usually have 18 tires. Suppose the probability that any one tire will blow out on a given cross-country trip is 0.03.
   a. Find the probability that none of the 18 tires blow out. 57.8%
   b. Find the probability that 1 of the tires blows out. 32.17%
   c. Find that probability that more than 2 of the tires blow out. 1.57%
   d. If a trucker wants to have a 95% probability of making the trip without a blowout, what must be the reliability of each tire? 99.72% reliability

Unit 7 Review

Perm & Comb.

1. ${}_{12}P_4 = \frac{12!}{(12-4)!} = \frac{12!}{8!} = 12 \cdot 11 \cdot 10 \cdot 9 = 11{,}880$

2. ${}_8P_5 = \frac{8!}{(8-5)!} = \frac{8!}{3!} = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 6{,}720$

3. ${}_{14}C_{11} = \frac{14!}{11!(14-11)!} = \frac{14!}{11!3!} = \frac{14 \cdot 13 \cdot 12}{3 \cdot 2 \cdot 1} = 364$

4. $\binom{n+4}{n+2} = \frac{(n+4)!}{(n+2)!(n+4-(n+2))!} = \frac{(n+4)!}{(n+2)!2!} = \frac{(n+4)(n+3)(n+2)!}{(n+2)!\,2!} = \frac{n^2+7n+12}{2}$

5. $\underset{T}{{}_{18}C_3} \cdot \underset{C}{4} \cdot \underset{S}{2} = \frac{18!}{15!3!} \cdot 8 = 6528$ pizzas

6. $\frac{15!}{4!2!2!} = 1362160800$ song orders

7. ${}_{26}P_3 = 26 \cdot 25 \cdot 24 = 15{,}600$ ways

8. $\frac{{}_4C_2 \cdot {}_4C_3}{{}_{52}C_5} = \frac{6 \cdot 4}{2{,}598{,}960} = \frac{24}{2598960} = \frac{1}{108290} = .0009\%$

9. ${}_{55}P_{10} = 55 \cdot 54 \cdot 53 \cdot 52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 = 1.06 \times 10^{17}$ ways

Expected Value

1. HHH HHT HTH THH TTH THT HTT TTT

P	2H = $\frac{3}{8}$	1H = $\frac{3}{8}$	3H/0H = $\frac{2}{8}=\frac{1}{4}$
V	\$5	\$3	−\$2

$EV = \frac{3}{8}(\$5) + \frac{3}{8}(\$3) + \frac{1}{4}(-\$2) = \$2.50$ win

2.

P	1st = $\frac{1}{20}$	2nd = $\frac{1}{10}$	3rd = $\frac{1}{4}$	$1-\frac{1}{20}-\frac{1}{10}-\frac{1}{4}=\frac{12}{20}=\frac{3}{5}$
V	4500	3500	1500	0

$EV = \frac{1}{20}(4500) + \frac{1}{10}(3500) + \frac{1}{4}(1500) + \frac{3}{5}(0)$

$= \$950$

No, you would lose \$50 on average

3.

P	1st = .002	2nd = .01	1 − .002 − .01 = .988
V	\$100	\$80	0

$EV = .002(100) + .01(80) + .988(0) = \$1$

The club should charge \$1.

4.

P	35%	40%	25%
V	-30K	0K	55K

$EV = .35(-30000) + .40(0) + .25(55000)$

$= \$3250$

You should invest because it is expected that you would make \$3250.

Binomial Distribution

1. $P(5) = {}_{12}C_5 (.4)^5 (.6)^7 = 22.7\%$

2. $P(X<7) = bcdf(20, .85, 6) = .0000046\%$

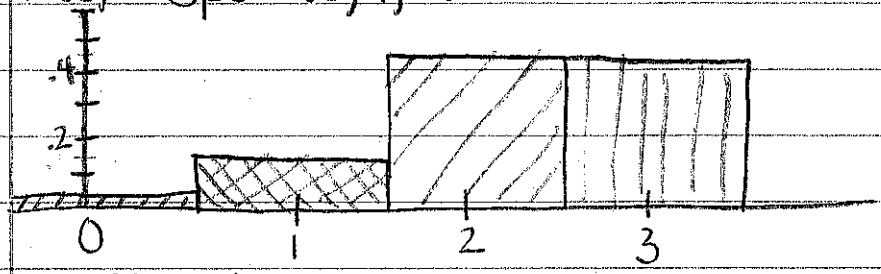
3. $P(X \geq 9) = 1 - bcdf(25, .35, 8) = 53.32\%$

4. $P(3<X<6) = bcdf(10, .42, 5) - bcdf(10, .42, 3) = 46.49\%$
   or $bpdf(10, .42, 4) + bpdf(10, .42, 5)$

5. $P(3 \leq X \leq 6) = bcdf(10, .42, 6) - bcdf(10, .42, 2) = 79.17\%$

6. $1 - \frac{1}{4} = \frac{3}{4}$ (dark hair)

   b. $P(0) = bpdf(3, \frac{3}{4}, 0) = 1.56\%$
   $P(1) = bpdf(3, \frac{3}{4}, 1) = 14.06\%$
   $P(2) = bpdf(3, \frac{3}{4}, 2) = 42.19\%$
   $P(3) = bpdf(3, \frac{3}{4}, 3) = 42.19\%$



   c. left-skew

7. $P(2) = bpdf(20, .08, 2) = 27.11\%$

b. $P(X > 5) = 1 - bcdf(20, .08, 5) = .38\%$

c. $P(X < 3) = bcdf(20, .08, 2) = 78.79\%$

d. $P(4 \leq X \leq 7) = bcdf(20, .08, 7) - bcdf(20, .08, 3)$
$= 7.05\%$

8. $P(0) = bpdf(18, .03, 0) = 57.8\%$

b. $P(1) = bpdf(18, .03, 1) = 32.17\%$

c. $P(X > 2) = 1 - bcdf(18, .03, 2) = 1.57\%$

d. $.95 = {}_{18}C_0 (p)^{18} (1-p)^0$

$.95 = 1(p)^{18}(1)$

$.95 = p^{18}$

$\sqrt[18]{.95} = p$

$p = .9972$

99.72% reliability per tire