Precalculus Midterm Review – Part 2

20. Use difference formulas to determine the exact values of $\sin(15^\circ)$, $\cos(15^\circ)$, and $\tan(15^\circ)$.

21. Use a sum formula to determine the exact value of $\tan 165^\circ$.

22. Justify. $(\tan x)(\csc x)(1-\sin^2 x) = \dfrac{1}{\sec x}$.

23. Simplify. $\cos\left(\dfrac{3\pi}{2}+\theta\right)$

24. Justify. $2\sin x(\sec x - \cos x) = 2\tan x - \sin 2x$

25. Solve $4\cos^2 x - 1 = 0$ on $0 \le x < 2\pi$.

26. Solve $3\tan x - \sqrt{3} = 0$.

27. Solve $\cot^2 \theta - 1 = 0$ on $0^\circ \le \theta < 180^\circ$.

28. Solve. $4\cos\theta\sin\theta + 2\cos\theta = 0$.

29. Determine $\sec 2A$ if $\sin A = \dfrac{6}{11}$ and angle A is in quadrant I.

30. Determine the exact value of $\cos(\alpha - \beta)$ if $\sin\alpha = \dfrac{7}{9}$, $0 \le \alpha < \dfrac{\pi}{2}$, and $\cos\beta = \dfrac{5}{13}$, $\dfrac{3\pi}{2} < \beta < 2\pi$.

Precalculus Midterm Review
Part 2

20)

$\sin 15° = \sin(150° - 135°) = \sin 150° \cos 135° - \cos 150° \sin 135°$

$= \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

$\cos 15° = \cos(225° - 210°) = \cos 225° \cos 210° + \sin 225° \cos 210°$

$= \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

$\tan 15° = \tan(45° - 30°)$

$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°} = \frac{1 - \left(\frac{1}{2}\right)\left(\frac{2}{\sqrt{3}}\right)}{1 + 1 \cdot \left(\frac{1}{2}\right)\left(\frac{2}{\sqrt{3}}\right)}$

$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}}{\sqrt{3}} - \frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \frac{\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)}{\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)} = \boxed{\frac{\sqrt{3}-1}{\sqrt{3}+1}}$

21) $\tan 165° = \tan(135° + 30°)$

$= \frac{\tan 135° + \tan 30°}{1 - \tan 135° \tan 30°} = \frac{(-1) + \frac{1}{\sqrt{3}}}{1 - (-1)\left(\frac{1}{\sqrt{3}}\right)}$

$= \frac{-\frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \frac{\left(\frac{1-\sqrt{3}}{\sqrt{3}}\right)}{\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)} = \boxed{\frac{1-\sqrt{3}}{1+\sqrt{3}}}$

22) $(\tan x)(\csc x)(1 - \sin^2 x)$

$= \left(\frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x}\right)\left(\frac{\cos^2 x}{1}\right) = \cos x = \frac{1}{\sec x}$

23) $\cos\left(\frac{3\pi}{2}+\theta\right)$

$= \cos\left(\frac{3\pi}{2}\right)\cos\theta - \sin\left(\frac{3\pi}{2}\right)\sin\theta$

$= (0)\cos\theta - (-1)\sin\theta = 0+\sin\theta = \boxed{\sin\theta}$

24) $2\sin x(\sec x - \cos x)$

$= 2\sin x\sec x - 2\sin x\cos x$

$= \frac{2\sin x}{1}\cdot\frac{1}{\cos x} - 2\sin x\cos x$

$= \frac{2\sin x}{\cos x} - 2\sin x\cos x = 2\tan x - \sin 2x$

25) $4\cos^2 x - 1 = 0 \qquad 0 \le x < 2\pi$

$(2\cos x+1)(2\cos x-1)=0$

$2\cos x+1=0 \quad 2\cos x-1=0$

$\cos x = -\frac{1}{2} \quad \cos x = \frac{1}{2}$

$x = 60°, 120°, 240°, 300°$

26) $3\tan x - \sqrt{3} = 0$

$\tan x = \frac{\sqrt{3}}{3}$

$\tan x = \frac{1}{\sqrt{3}} \rightarrow$ note $\frac{1}{\sqrt{3}} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{1}{2}\div\frac{\sqrt{3}}{2}$

$\tan x = \frac{(1/2)}{(\sqrt{3}/2)}$

so where does $\sin x = \frac{1}{2}$ and $\cos x = \frac{\sqrt{3}}{2}$

or where $\sin x = -\frac{1}{2}$ and $\cos x = -\frac{\sqrt{3}}{2}$?

$30°, 210°$, etc. $\boxed{30° + n\cdot 180°}$

27) $\cot^2\theta - 1 = 0$ on $0° \le \theta < 180°$

$\cot^2\theta = 1$

$\cot\theta = \pm 1$

where does $\frac{\cos\theta}{\sin\theta} = \pm 1$? (45° and 135°)

28) $4\cos\theta\sin\theta + 2\cos\theta = 0$

$2\cos\theta(2\sin\theta + 1) = 0$

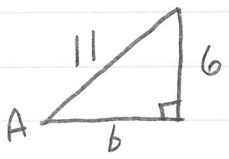
$2\cos\theta = 0$ $\quad$ $2\sin\theta + 1 = 0$

$\cos\theta = 0$ $\quad$ $\sin\theta = -\frac{1}{2}$

$\theta = 90° + n\cdot 180°$, $\quad \theta = 210° + n\cdot 360°$

$\theta = 330° + n\cdot 360°$

29) $\sin A = \frac{6}{11}$



$6^2 + b^2 = 11^2$

$b^2 = 11^2 - 6^2$

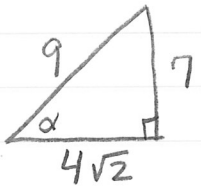
$b = \sqrt{85}$

thus $\cos A = \frac{\sqrt{85}}{11}$

now $\sin 2A = 2\sin A\cos A$

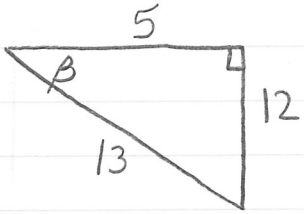
$= 2\left(\frac{6}{11}\right)\left(\frac{\sqrt{85}}{11}\right) = \frac{12\sqrt{85}}{121}$

30) $\sin\alpha = \frac{7}{9}$ $\quad 0 < \alpha < \frac{\pi}{2}$



$\sqrt{9^2 - 7^2} = \sqrt{32} = 4\sqrt{2}$

$\cos\alpha = \frac{4\sqrt{2}}{9}$



$\sin\beta = \frac{12}{13}$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \left(\frac{4\sqrt{2}}{9}\right)\left(\frac{5}{13}\right) + \left(\frac{7}{9}\right)\left(\frac{12}{13}\right)$

$= \frac{20\sqrt{2}}{117} + \frac{84}{117} = \boxed{\frac{84 + 20\sqrt{2}}{117}}$