Part 1

a) $\frac{1}{\cos x} = \sec x$

b) $\frac{1}{\sec x} = \cos x$

c) $\frac{\cos x}{\sin x} = \cot x$

d) $\csc\left(\frac{\pi}{2} - x\right) = \frac{1}{\sin\left(\frac{\pi}{2} - x\right)}$

$\frac{1}{\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x} = \frac{1}{1\cdot\cos x} = \sec x$

e) $\sec(-x) = \sec x$

f) $\frac{1}{\cot^2 x + 1} = \frac{1}{\csc^2 x} = \sin^2 x$

g) $\sin^2\alpha - \cos^2\alpha$

$\sin^2\alpha - \sin\alpha\cos\alpha$

$(\sin\alpha - \cos\alpha)(\sin\alpha + \cos\alpha)$

$\sin\alpha(\sin\alpha - \cos\alpha)$

$\frac{\sin\alpha + \cos\alpha}{\sin\alpha} = \frac{\sin}{\sin} + \frac{\cos}{\sin}$

$1 + \cot\alpha$

h) $\tan\left(\frac{\pi}{2} - x\right)\sec x$

$\frac{\sin(\pi/2 - x)}{\cos(\pi/2 - x)} \cdot \frac{1}{\cos x}$

$\frac{\sin(\pi/2)\cos x - \cos\pi/2\sin x}{\cos(\pi/2)\cos x + \sin(\pi/2)\sin x} \cdot \frac{1}{\cos x}$

$\frac{1\cdot\cos x - 0}{0\cdot\cos x + 1\cdot\sin x} = \frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}$

$= \frac{1}{\sin x} = \csc x$

i) $\tan^2\theta(\csc^2\theta - 1)$

$\tan^2\theta(1 + \cot^2\theta - 1)$

$\tan^2\theta \cdot \cot^2\theta$

$= 1$

Part 2

a) $\cos x(\tan^2 x + 1) = \sec x$

$\cos x(\sec^2 x - 1 + 1) = \sec x$

$\frac{1}{\sec x}(\sec^2 x) = \sec x$

$\sec x = \sec x$

b) $\sin^3\theta + \sin\theta\cos^2\theta = \sin\theta$

$\sin\theta(\sin^2\theta + \cos^2\theta) = \sin\theta$

$\sin\theta(1) = \sin\theta$

$\sin\theta = \sin\theta$

c) $\sin^5 x\cos^2 x = (\cos^2 x - 2\cos^4 x + \cos^6 x)\sin x$

$= \cos^2 x(1 - 2\cos^2 x + \cos^4 x)\sin x$

$= \cos^2 x(1 - \cos^2 x)(1 - \cos^2 x)\sin x$

$= \cos^2 x(\sin^2 x)(\sin^2 x)\sin x$

$\sin^5 x\cos^2 x = \sin^5 x\cos^2 x$

d) $\frac{\csc(-x)}{\sec(-x)} = -\cot(x)$

$\frac{1}{\sin(-x)} \cdot \frac{\cos(-x)}{1} =$

$\frac{+\cos(x)}{-\sin(x)} \Rightarrow -\cot(x) = -\cot x$

e) $\sin^2 x + \sin^2\left(\frac{\pi}{2} - x\right) = 1$

$\sin^2 x + \sin^2\left(\frac{\pi}{2}\right)\cos^2 x - \cos^2\frac{\pi}{2}\sin^2 x = 1$

$\sin^2 x + 1\cdot\cos^2 x - 0\cdot\sin^2 x = 1$

$\sin^2 x + \cos^2 x = 1$

$1 = 1$

Part 2 cont'd

f) $\cos(x+\frac{\pi}{2}) = -\sin x$

$\cos x \cdot \cos\frac{\pi}{2} - \sin x \cdot \sin\frac{\pi}{2} = -\sin x$

$(\cos x)\cdot 0 - \sin x \cdot (1) = -\sin x$

$-\sin x = -\sin x$

g) $\cot\left(\frac{\pi}{2}-x\right) = \tan x$

$\frac{\cos(\pi/2 - x)}{\sin(\pi/2 - x)} = \tan x$

$\frac{\cos\pi/2\cos x + \sin\pi/2\sin x}{\sin\pi/2\cos x - \cos\pi/2\sin x} = \tan x$

$\frac{\sin x}{\cos x} = \tan x$

$\tan x = \tan x$

h) $6\sin x\cos x = 3\sin 2x$

$6\sin x\cos x = 3(2\sin x\cos x)$

$6\sin x\cos x = 6\sin x\cos x$

Part 3

a) $2\sin x - 1 = 0$

$2\sin x = 1$

$\sin x = 1/2 \quad \} x = \sin^{-1}(1/2)$

[a] $x = \pi/6, 5\pi/6 \quad [0, 2\pi)$

[b] $x = \pi/6 + 2\pi k \} (-\infty, \infty)$

$x = 5\pi/6 + 2\pi k \}$

b) $\sin x = \sqrt{3} - \sin x$

$2\sin x = \sqrt{3}$

$\sin x = \sqrt{3}/2$

[a] $x = \pi/3, 2\pi/3 \quad [0, 2\pi)$

[b] $x = \pi/3 + 2\pi k \} (-\infty, \infty)$

$x = 2\pi/3 + 2\pi k \}$

c) $3\sqrt{3}\tan x = 3$

$\tan x = \frac{3}{3\sqrt{3}}$

$\tan x = \frac{1}{\sqrt{3}}$

$\frac{\sin x}{\cos x} = \frac{1/2}{\sqrt{3}/2}$

[a] $x = \frac{\pi}{6}, \frac{7\pi}{6} \quad [0, 2\pi)$

[b] $x = \frac{\pi}{6} + \pi k \} (-\infty, \infty)$

d) $3\csc^2 x = 4$

$\csc^2 x = \frac{4}{3}$

$\frac{1}{\sin^2 x} = \frac{4}{3}$

$\frac{1}{\sin x} = \pm\frac{2}{\sqrt{3}}$

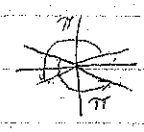
$\sin x = \pm\sqrt{3}/2$

[a] $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \quad [0, 2\pi)$

[b] $x = \frac{\pi}{3} + \pi k \} (-\infty, \infty)$

$x = \frac{2\pi}{3} + \pi k \}$

e) $4\cos^2 x - 3 = 0$



$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

[a] $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \quad [0, 2\pi)$

[b] $x = \frac{\pi}{6} + \pi k \} (-\infty, \infty)$

$x = \frac{5\pi}{6} + \pi k \}$

f) $\sin x - \tan x = 0$

$\cos x\left[\sin x - \frac{\sin x}{\cos x} = 0\right]\cos x$

$\sin x\cdot\cos x - \sin x = 0$

$(\sin x)(\cos x - 1) = 0$

$\sin x = 0 \quad \{ \cos x - 1 = 0$

$x = 0, \pi \quad \{ \cos x = 1$

$x = 0$

[a] $x = 0, \pi$

[b] $x = 0 + \pi k$

$\sin(0) - \tan(0) = 0$

$0 - \frac{0}{1} = 0$

$\sin(\pi) - \tan(\pi) = 0$

$0 - \frac{0}{-1} = 0$

Part 3 cont'd

$\frac{2\pi}{8} = \frac{\pi}{4}$

g) $2\cos^2 x - \cos x = 1$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0 \quad \cos x - 1 = 0$

$\cos x = -\frac{1}{2} \quad \cos x = 1$

a] $x = \frac{2\pi}{3}, \frac{4\pi}{3} \quad x = 0 \} [0, 2\pi)$

b] $x = 0 + 2\pi k$

$x = \frac{2\pi}{3} + 2\pi k \} (-\infty, \infty)$

$x = \frac{4\pi}{3} + 2\pi k$

h) $\cos^2 x + \sin x = 1$

$1 - \sin^2 x + \sin x = 1$

$0 = \sin^2 x - \sin x$

$0 = \sin x (\sin x - 1)$

$\sin x = 0 \quad \sin x - 1 = 0$

$\sin x = 1$

a] $x = 0, \pi \quad x = \pi/2 \} [0, 2\pi)$

b] $x = \pi k \} (-\infty, \infty)$

$x = \frac{\pi}{2} + 2\pi k$

i) $2\sin 2x - \sqrt{2} = 0$

$2\sin 2x = \sqrt{2}$

$\sin 2x = \sqrt{2}/2$

$2x = \frac{\pi}{4} \quad 2x = \frac{3\pi}{4} \quad 2x = \frac{9\pi}{4} \quad 2x = \frac{11\pi}{4}$

a] $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8} \} [0, 2\pi)$

b] $x = \frac{\pi}{8} + \pi k \} (-\infty, \infty)$

$x = \frac{3\pi}{8} + \pi k$

j) $\cos 4x (\cos x - 1) = 0$

$\cos 4x = 0 \quad \cos x - 1 = 0$

$4x = \frac{\pi}{2} \quad 4x = \frac{3\pi}{2} \quad \cos x = 1$

$4x = \frac{5\pi}{2} \quad 4x = \frac{7\pi}{2} \quad x = 0$

$4x = \frac{9\pi}{2} \quad 4x = \frac{11\pi}{2}$

$4x = \frac{13\pi}{2} \quad 4x = \frac{15\pi}{2}$

divide by 4 ↓

a] $x = 0, \pi/8, 3\pi/8, 5\pi/8, 7\pi/8, 9\pi/8, 11\pi/8, 13\pi/8, 15\pi/8 \} [0, 2\pi)$

b] $x = 2\pi k$

$x = \pi/8 + \frac{\pi}{4} k \} (-\infty, \infty)$

k) $\cos 4x - 7\cos 2x = 8$ Note: $\cos 2A = 2\cos^2 A - 1$

$\cos 2(2x) - 7\cos 2x = 8$

$2\cos^2 2(x) - 1 - 7\cos 2x - 8 = 0$

$2\cos^2 2x - 7\cos 2x - 9 = 0$

$(2\cos 2x - 9)(\cos 2x + 1) = 0$

$2\cos 2x - 9 = 0 \quad \cos 2x + 1 = 0$

$2\cos 2x = 9 \quad \cos 2x = -1$

$\cos 2x = 9/2 \quad 2x = \pi, 3\pi$

fails ≤ 1

a] $x = \pi/2, 3\pi/2$ $[0, 2\pi)$

b] $x = \frac{\pi}{2} + \pi k$

$(-\infty, \infty)$

l) $\sin^2 x - 2\sin x = 0$

$\sin x(\sin x - 2) = 0$

$\sin x = 0$ $\quad \underbrace{\sin x = 2}_{\text{fail}}$

a] $x = 0, \pi$ $\}[0, 2\pi)$

b] $x = \pi \cdot k$ $\}(-\infty, \infty)$ or $x = 0 + \pi k$

m) $\tan^2\theta + \tan\theta - 12 = 0$

$(\tan\theta + 4)(\tan\theta - 3) = 0$

$\tan\theta = -4 \quad \tan\theta = 3$

must use calculator

$\theta = \tan^{-1}(-4) \quad \theta = \tan^{-1}(3)$

$\theta = -1.3258 \quad \theta = 1.2490$

ref $\angle$'s $\quad \& 4.3906$

$\theta = 1.8158$

$\theta = 4.9574$

a] $\theta = 1.2490, 1.8158$ $\}[0, 2\pi)$
$4.3906, 4.9574$

b] $\theta = 1.2490 + \pi k$ $\}(-\infty, \infty)$
$\theta = 1.8158 + \pi k$

n) $\sin\left(x + \frac{\pi}{2}\right) - \sin\left(x - \frac{\pi}{2}\right) = \sqrt{3}$

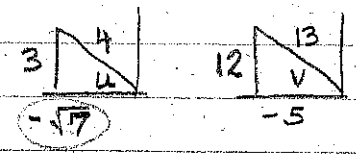
- $\sin x\cos\frac{\pi}{2} + \cos x\sin\left(\frac{\pi}{2}\right) - \left[\sin x\cos\frac{\pi}{2} - \cos x\sin\frac{\pi}{2}\right] = \sqrt{3}$
- $\sin x(0) + \cos x(1) - \sin x(0) + \cos x(1) = \sqrt{3}$
- $2\cos x = \sqrt{3}$
- $\cos x = \sqrt{3}/2$

a] $x = \cos^{-1}(\sqrt{3}/2) = \pi/6, 11\pi/6$ $[0, 2\pi)$

b] $x = \pi/6 + 2\pi k$ $\}(-\infty, \infty)$
$x = 11\pi/6 + 2\pi k$

4/5

4a] $\sin u = 3/4$, $\cos v = -5/13$ $\}$ QII



i] $\sin(u+v)$

$\sin u\cos v + \cos u\sin v$

$\left(\frac{3}{4}\right)\left(\frac{-5}{13}\right) + \left(\frac{-\sqrt{7}}{4}\right)\left(\frac{12}{13}\right)$

$= \frac{-15 - 12\sqrt{7}}{52}$

ii] $\cos(u-v)$

$\cos u\cos v + \sin u\sin v$

$\left(\frac{-\sqrt{7}}{4}\right)\left(\frac{-5}{13}\right) + \left(\frac{3}{4}\right)\left(\frac{12}{13}\right)$

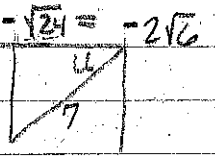
$= \frac{5\sqrt{7} + 36}{52}$

iii] $\cos(u+v)$

$\cos u\cos v - \sin u\sin v$

$= \frac{5\sqrt{7} - 36}{52}$

4b) $\sin u = -5/7 \quad \pi < u < 3\pi/2$



i) $\sin 2u = 2\sin u \cos u$

$= 2(-5/7)(-2\sqrt{6}/7)$

$= 20\sqrt{6}/49$

ii) $\cos 2u = \cos^2 u - \sin^2 u$

$= \left(\frac{-2\sqrt{6}}{7}\right)^2 - \left(\frac{-5}{7}\right)^2$

$= \frac{24}{49} - \frac{25}{49}$

$= -1/49$

iii) $\tan 2u = \frac{\sin 2u}{\cos 2u}$

(from above)

$$\frac{\frac{20\sqrt{6}}{49}}{-\frac{1}{49}} \cdot \frac{\frac{49}{1}}{\frac{49}{1}}$$

$= -20\sqrt{6}$